Designation: C 885 – 87 (Reapproved 2002)

Standard Test Method for Young's Modulus of Refractory Shapes by Sonic Resonance[1]

This standard is issued under the fixed designation C 885; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This test method covers a procedure for measuring the resonance frequency in the flexural (transverse) mode of vibration of rectangular refractory brick or rectangularly shaped monoliths at room temperature. Young's modulus is calculated from the resonance frequency of the shape, its mass (weight) and dimensions.

1.2 *This standard does not purport to address all of the safety concerns, if any,associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

C 134 Test Methods for Size, Dimensional Measurements, and Bulk Density of Refractory Brick and Insulating Firebrick[2]

C 215 Test Method for Fundamental Transverse, Longitudinal, and Torsional Frequencies of Concrete Specimens[3]

C 623 Test Method for Young's Modulus, Shear Modulus, and Poisson's Ratio for Glass and Glass-Ceramics by Resonance[4]

C 747 Test Method for Moduli of Elasticity and Fundamental Frequencies of Carbon and Graphite Materials by Sonic Resonance[2]

C 848 Test Method for Young's Modulus, Shear Modulus, and Poisson's Ratio for Ceramic Whitewares by Resonance[4]

3. Summary of Test Method

3.1 Test specimens are vibrated in flexure over a broad frequency range; mechanical excitation is provided through the use of a vibrating driver that transforms an initial electrical signal into a mechanical vibration. A detector senses the resulting mechanical vibrations of the specimen and transforms them into an electrical signal that can be displayed on the screen of an oscilloscope to detect resonance by a Lissajous figure. The calculation of Young's modulus from the resonance frequency measured is simplified by assuming that Poisson's ratio is $\frac{1}{6}$ for all refractory materials.

4. Significance and Use

4.1 Young's modulus is a fundamental mechanical property of a material.

4.2 This test method is used to determine the dynamic modulus of elasticity of rectangular shapes. Since the test is nondestructive, specimens may be used for other tests as desired.

4.3 This test method is useful for research and development, engineering application and design, manufacturing process control, and for developing purchasing specifications.

4.4 The fundamental assumption inherent in this test method is that a Poisson's ratio of $\frac{1}{6}$ is typical for heterogeneous refractory materials. The actual Poisson's ratio may differ.

5. Apparatus

5.1 A block diagram of a suggested test apparatus arrangement is shown in Fig. 1. Details of the equipment are as follows:

5.1.1 *Audio Oscillator*, having a continuously variable calibrated-frequency output from about 50 Hz to at least 10 kHz.

5.1.2 *Audio Amplifier*, having a power output sufficient to ensure that the type of driver used can excite the specimen; the output of the amplifier must be adjustable.

5.1.3 *Driver*, which may consist of a transducer or a loudspeaker from which the cone has been removed and replaced with a probe (connecting rod) oriented parallel to the direction of the vibration; suitable vibration-isolating mounts.

NOTE 1—For small specimens, an air column may preferably be used for "coupling" the loudspeaker to the specimen.

[1] This test method is under the jurisdiction of ASTM Committee C-8 on Refractories, and is the direct responsibility of Subcommittee C08.01 on Strength Properties.

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[2] *Annual Book of ASTM Standards*, Vol 15.01.

[3] *Annual Book of ASTM Standards*, Vol 04.02.

[4] *Annual Book of ASTM Standards*, Vol 15.02.

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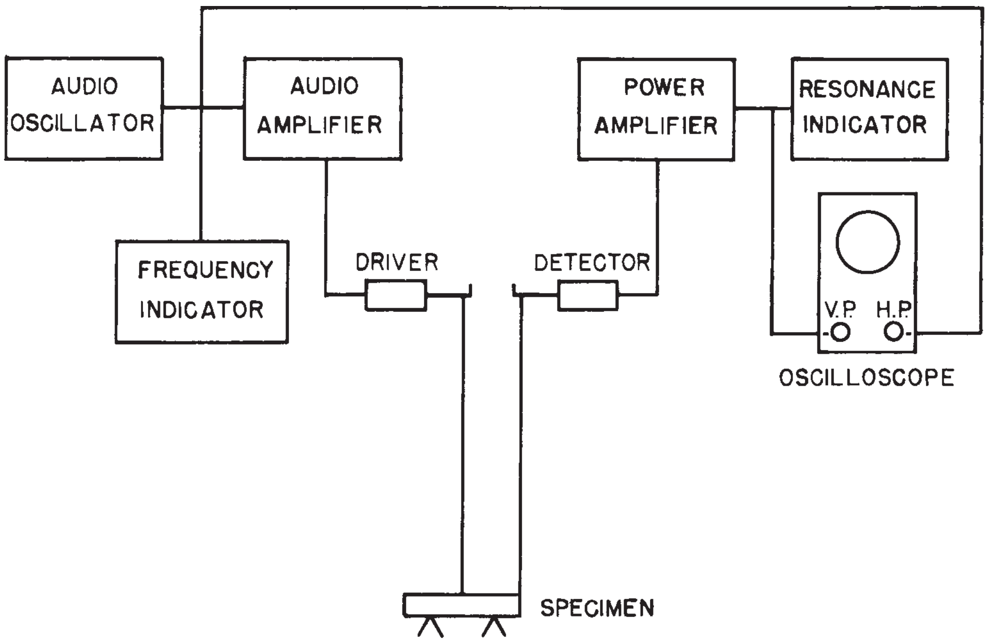

FIG. 1 Block Diagram of Apparatus

5.1.4 *Detector*, which may be a transducer or a balance-mounted monaural (crystal or magnetic) phonograph pick-up cartridge of good frequency response; the detector should be movable across the specimen; suitable vibration-isolating mounts.

5.1.5 *Pre-Scope Amplifier* in the detector circuit, impedance-matched with the detector used; the output must be adjustable.

5.1.6 *Indicating Devices*, including an oscilloscope, a resonance indicator (voltmeter or ammeter), and a frequency indicator, which may be the control dial of the audio-oscillator (accurately readable to ±30 Hz or better) or, preferably, a frequency meter, for example, a digital frequency counter.

5.1.7 *Specimen Support*, consisting of two knife edges (can be steel, rubber-coated steel, or medium-hard rubber) of a length at least equal to the width of the specimens; the distance between the knife edges must be adjustable.

NOTE 2—The support for the knife edges may be a foam rubber pad, and should be vibration-isolated from drive and detector supports.

NOTE 3—Alternatively, knife edges can be omitted and the specimen may be placed directly on a foam rubber pad if the test material is easily excitable due to its composition and geometry.

6. Sampling and Specimen Preparation

6.1 Specimens must be rectangular prisms. They may be full straight brick or rectangular samples cut from brick shapes; rectangular straight shapes of monolithic refractories, or rectangular specimens cut from monolithic shapes. For best results, their length to thickness ratio should be at least 3 to 1. Maximum specimen size and mass are primarily determined by the test system's energy capability and by the resonance response characteristics of the material. Minimum specimen size and mass are primarily determined by adequate and optimum coupling of the driver and the detector to the specimen, and by the resonance response characteristics of the material. Measure the mass (weight) and dimensions of the dry specimens in accordance with Test Methods C 134 and record.

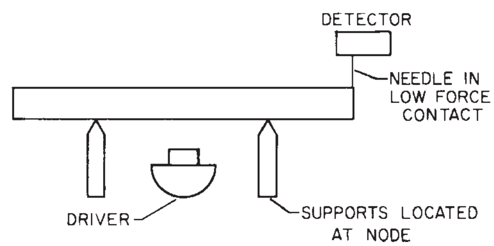

FIG. 2 Typical Specimen Positioning for Measurement of Flexural Resonance

7. Procedure

7.1 Refractories can vary markedly in their response to the driver's frequency; the geometry of the specimens also plays a significant role in their response characteristics. Variations in the following procedure are permissible as long as flexural and fundamental resonance are verified (Note 6 and Note 7). Fig. 2 and Fig. 3 illustrate a typical specimen positioning and the desired mode of vibration, respectively.

7.2 *Sample Placement*—Place the specimen "flat" (thickness dimension perpendicular to supports) on parallel knife edges at 0.224 *l* (where *l* is the length of the specimen) from its ends. Optionally, the specimen can be placed on a foam rubber pad.

7.3 *Driver Placement*—Place the driver preferably at the center of the top or bottom face of the specimen using moderate balanced pressure or spring action.

NOTE 4—Especially with small (thin) specimens, the lightest possible driver pressure to ensure adequate "coupling" must be used in order to achieve proper resonance response. In small specimens, exact placement

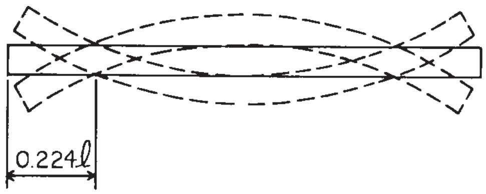

FIG. 3 Fundamental Mode of Vibration in Flexure (Side View)

of the driver at the very center of the flat specimen is important; also, an air column may be used for "coupling."

7.4 *Detector Placement*—Place the detector preferably at one end of the specimen and at the center of either the width or thickness (considering the orientation of maximum response of the detector) using minimal pressure.

NOTE 5—Make sure that the stylus of the phonograph cartridge (if used) is well secured.

7.5 Activate and warm up the equipment so that power adequate to excite the specimen is delivered to the driver. Set the gain on the detector circuit high enough to detect vibration in the specimen, and to display it on the oscilloscope screen with sufficient amplitude to measure accurately the frequency at which the signal amplitude is maximized. Adjust the oscilloscope so that a sharply defined horizontal baseline exists when the specimen is not excited. Scan frequency with the audio oscillator until fundamental flexural specimen resonance is indicated by an oval to circular Lissajous figure at the oscilloscope and maximum output is shown at the resonance indicator. Record the resonance frequency.

NOTE 6—To verify the flexural mode of vibration, move the detector to the top center of the specimen. The oval or circular oscilloscope pattern shall be maintained. Placement of the detector above the nodal points (at 0.224 *l*) shall cause a Lissajous pattern and high output at the resonance indicator to disappear.

NOTE 7—To verify the fundamental mode of flexural resonance, excite the specimen at one half of the frequency established in 7.5. A "figure eight" Lissajous pattern should appear at the oscilloscope when the detector is placed at the end center or at the top center of the specimen.

8. Calculation

8.1 Data determined on individual specimens include:

8.1.1 l = length of specimen, in.,

8.1.2 b = width of specimen, in.,

8.1.3 t = thickness of specimen, in.,

8.1.4 w = mass (weight) of specimen, lb, and

8.1.5 f = fundamental flexural resonance frequency, Hz.

8.2 Calculate Young's modulus *E*, in psi, of the specimen as follows:

$$E = C_1 \cdot w \cdot f^2 \tag{1}$$

where $C_1 = [C_1 b]/b$ (in s ^{2} /in. ^{2}) is calculated from values of $[C_1 b]$ listed in Table 1 for various *l/t* ratios based on Pickett's[5] equations solved for a Poisson's ratio of ⅙ . Alternatively, $[C_1 b]$ can be computed directly from l and t using Pickett's original equations and correction factors, as described in Appendix X1.

[5] Pickett, G., "Equations for Computing Elastic Constants from Flexural and Torsional Resonant Frequencies of Vibration of Prisms and Cylinders," *Proceedings*, ASTM, Vol 45, 1945, pp. 846–863.

TABLE 1 $[C_1 b]$ Values

l/t	$[C_1 b]$	l/t	$[C_1 b]$	l/t	$[C_1 b]$	l/t	$[C_1 b]$	l/t	$[C_1 b]$	l/t	$[C_1 b]$
2.50	0.0750	3.10	0.1200	3.70	0.1815	4.30	0.2627	4.90	0.3665	5.50	0.4963
2.51	0.0756	3.11	0.1209	3.71	0.1827	4.31	0.2642	4.91	0.3685	5.51	0.4988
2.52	0.0763	3.12	0.1218	3.72	0.1839	4.32	0.2657	4.92	0.3704	5.52	0.5012
2.53	0.0769	3.13	0.1227	3.73	0.1851	4.33	0.2673	4.93	0.3724	5.53	0.5036
2.54	0.0776	3.14	0.1236	3.74	0.1863	4.34	0.2688	4.94	0.3743	5.54	0.5060
2.55	0.0782	3.15	0.1245	3.75	0.1875	4.35	0.2704	4.95	0.3763	5.55	0.5084
2.56	0.0789	3.16	0.1254	3.76	0.1887	4.36	0.2720	4.96	0.3783	5.56	0.5109
2.57	0.0795	3.17	0.1263	3.77	0.1899	4.37	0.2735	4.97	0.3803	5.57	0.5133
2.58	0.0802	3.18	0.1272	3.78	0.1911	4.38	0.2751	4.98	0.3823	5.58	0.5158
2.59	0.0808	3.19	0.1281	3.79	0.1924	4.39	0.2767	4.99	0.3843	5.59	0.5183
2.60	0.0815	3.20	0.1291	3.80	0.1936	4.40	0.2783	5.00	0.3863	5.60	0.5207
2.61	0.0822	3.21	0.1300	3.81	0.1948	4.41	0.2799	5.01	0.3883	5.61	0.5232
2.62	0.0828	3.22	0.1309	3.82	0.1961	4.42	0.2815	5.02	0.3903	5.62	0.5257
2.63	0.0835	3.23	0.1318	3.83	0.1973	4.43	0.2831	5.03	0.3924	5.63	0.5282
2.64	0.0842	3.24	0.1328	3.84	0.1986	4.44	0.2847	5.04	0.3944	5.64	0.5307
2.65	0.0849	3.25	0.1337	3.85	0.1999	4.45	0.2864	5.05	0.3964	5.65	0.5332
2.66	0.0856	3.26	0.1347	3.86	0.2011	4.46	0.2880	5.06	0.3985	5.66	0.5358
2.67	0.0863	3.27	0.1356	3.87	0.2024	4.47	0.2896	5.07	0.4005	5.67	0.5383
2.68	0.0870	3.28	0.1366	3.88	0.2037	4.48	0.2913	5.08	0.4026	5.68	0.5408
2.69	0.0877	3.29	0.1376	3.89	0.2050	4.49	0.2929	5.09	0.4047	5.69	0.5434
2.70	0.0884	3.30	0.1385	3.90	0.2062	4.50	0.2946	5.10	0.4068	5.70	0.5459
2.71	0.0891	3.31	0.1395	3.91	0.2075	4.51	0.2963	5.11	0.4089	5.71	0.5485
2.72	0.0898	3.32	0.1405	3.92	0.2088	4.52	0.2979	5.12	0.4110	5.72	0.5511
2.73	0.0905	3.33	0.1415	3.93	0.2101	4.53	0.2996	5.13	0.4131	5.73	0.5537
2.74	0.0912	3.34	0.1425	3.94	0.2115	4.54	0.3013	5.14	0.4152	5.74	0.5562
2.75	0.0920	3.35	0.1435	3.95	0.2128	4.55	0.3030	5.15	0.4173	5.75	0.5588
2.76	0.0927	3.36	0.1445	3.96	0.2141	4.56	0.3047	5.16	0.4194	5.76	0.5615
2.77	0.0934	3.37	0.1455	3.97	0.2154	4.57	0.3064	5.17	0.4216	5.77	0.5641
2.78	0.0942	3.38	0.1465	3.98	0.2168	4.58	0.3081	5.18	0.4237	5.78	0.5667
2.79	0.0949	3.39	0.1475	3.99	0.2181	4.59	0.3098	5.19	0.4258	5.79	0.5693

TABLE 1 *Continued*

l/t	[C_1b]	l/t	[C_1b]	l/t	[C_1b]	l/t	[C_1b]	l/t	[C_1b]	l/t	[C_1b]
2.80	0.0957	3.40	0.1485	4.00	0.2194	4.60	0.3116	5.20	0.4280	5.80	0.5720
2.81	0.0964	3.41	0.1496	4.01	0.2208	4.61	0.3133	5.21	0.4302	5.81	0.5746
2.82	0.0972	3.42	0.1506	4.02	0.2222	4.62	0.3150	5.22	0.4323	5.82	0.5773
2.83	0.0979	3.43	0.1516	4.03	0.2235	4.63	0.3168	5.23	0.4345	5.83	0.5799
2.84	0.0987	3.44	0.1527	4.04	0.2249	4.64	0.3185	5.24	0.4367	5.84	0.5826
2.85	0.0994	3.45	0.1537	4.05	0.2263	4.65	0.3203	5.25	0.4389	5.85	0.5853
2.86	0.1002	3.46	0.1548	4.06	0.2277	4.66	0.3220	5.26	0.4411	5.86	0.5880
2.87	0.1010	3.47	0.1558	4.07	0.2290	4.67	0.3238	5.27	0.4433	5.87	0.5907
2.88	0.1018	3.48	0.1569	4.08	0.2304	4.68	0.3256	5.28	0.4455	5.88	0.5934
2.89	0.1026	3.49	0.1579	4.09	0.2318	4.69	0.3274	5.29	0.4478	5.89	0.5961
2.90	0.1033	3.50	0.1590	4.10	0.2332	4.70	0.3292	5.30	0.4500	5.90	0.5989
2.91	0.1041	3.51	0.1601	4.11	0.2347	4.71	0.3310	5.31	0.4522	5.91	0.6016
2.92	0.1049	3.52	0.1612	4.12	0.2361	4.72	0.3328	5.32	0.4545	5.92	0.6043
2.93	0.1057	3.53	0.1623	4.13	0.2375	4.73	0.3346	5.33	0.4568	5.93	0.6071
2.94	0.1065	3.54	0.1633	4.14	0.2389	4.74	0.3364	5.34	0.4590	5.94	0.6099
2.95	0.1074	3.55	0.1644	4.15	0.2404	4.75	0.3383	5.35	0.4613	5.95	0.6126
2.96	0.1082	3.56	0.1655	4.16	0.2418	4.76	0.3401	5.36	0.4636	5.96	0.6154
2.97	0.1090	3.57	0.1667	4.17	0.2433	4.77	0.3419	5.37	0.4659	5.97	0.6182
2.98	0.1098	3.58	0.1678	4.18	0.2447	4.78	0.3438	5.38	0.4682	5.98	0.6210
2.99	0.1106	3.59	0.1689	4.19	0.2462	4.79	0.3456	5.39	0.4705	5.99	0.6238
3.00	0.1115	3.60	0.1700	4.20	0.2476	4.80	0.3475	5.40	0.4728	6.00	0.6266
3.01	0.1123	3.61	0.1711	4.21	0.2491	4.81	0.3494	5.41	0.4751	6.01	0.6294
3.02	0.1131	3.62	0.1723	4.22	0.2506	4.82	0.3513	5.42	0.4774	6.02	0.6323
3.03	0.1140	3.63	0.1734	4.23	0.2521	4.83	0.3531	5.43	0.4798	6.03	0.6351
3.04	0.1148	3.64	0.1746	4.24	0.2536	4.84	0.3550	5.44	0.4821	6.04	0.6380
3.05	0.1157	3.65	0.1757	4.25	0.2551	4.85	0.3569	5.45	0.4845	6.05	0.6408
3.06	0.1166	3.66	0.1769	4.26	0.2566	4.86	0.3588	5.46	0.4868	6.06	0.6437
3.07	0.1174	3.67	0.1780	4.27	0.2581	4.87	0.3608	5.47	0.4892	6.07	0.6466
3.08	0.1183	3.68	0.1792	4.28	0.2596	4.88	0.3627	5.48	0.4916	6.08	0.6495
3.09	0.1192	3.69	0.1804	4.29	0.2611	4.89	0.3646	5.49	0.4940	6.09	0.6524
6.10	0.6553	6.40	0.7466	6.70	0.8465	7.00	0.9552	8.30	1.5383	9.75	2.4336
6.11	0.6582	6.41	0.7498	6.71	0.8499	7.05	0.9742	8.35	1.5647	9.80	2.4696
6.12	0.6611	6.42	0.7530	6.72	0.8534	7.10	0.9934	8.40	1.5913	9.85	2.5059
6.13	0.6640	6.43	0.7562	6.73	0.8569	7.15	1.0130	8.45	1.6183	9.90	2.5427
6.14	0.6670	6.44	0.7594	6.74	0.8604	7.20	1.0327	8.50	1.6455	9.95	2.5797
6.15	0.6699	6.45	0.7627	6.75	0.8640	7.25	1.0528	8.55	1.6731	10.00	2.6172
6.16	0.6729	6.46	0.7659	6.76	0.8675			8.60	1.7010		
6.17	0.6758	6.47	0.7692	6.77	0.8710			8.65	1.7292		
6.18	0.6788	6.48	0.7724	6.78	0.8746			8.70	1.7578		
6.19	0.6818	6.49	0.7757	6.79	0.8781						
								8.75	1.7866		
6.20	0.6848	6.50	0.7789	6.80	0.8817	7.30	1.0731	8.80	1.8158		
6.21	0.6878	6.51	0.7822	6.81	0.8853	7.35	1.0937	8.85	1.8453		
6.22	0.6908	6.52	0.7855	6.82	0.8889	7.40	1.1146	8.90	1.8751		
6.23	0.6938	6.53	0.7888	6.83	0.8925	7.45	1.1357	8.95	1.9052		
6.24	0.6969	6.54	0.7921	6.84	0.8961	7.50	1.1571	9.00	1.9357		
6.25	0.6999	6.55	0.7955	6.85	0.8997	7.55	1.1788	9.05	1.9665		
6.26	0.7030	6.56	0.7988	6.86	0.9033	7.60	1.2007	9.10	1.9977		
6.27	0.7060	6.57	0.8021	6.87	0.9069	7.65	1.2230	9.15	2.0291		
6.28	0.7091	6.58	0.8055	6.88	0.9106	7.70	1.2455	9.20	2.0609		
6.29	0.7122	6.59	0.8088	6.89	0.9143	7.75	1.2683				
6.30	0.7153	6.60	0.8122	6.90	0.9179	7.80	1.2914	9.25	2.0931		
6.31	0.7183	6.61	0.8156	6.91	0.9216	7.85	1.3148	9.30	2.1256		
6.32	0.7215	6.62	0.8190	6.92	0.9253	7.90	1.3384	9.35	2.1584		
6.33	0.7246	6.63	0.8224	6.93	0.9290	7.95	1.3624	9.40	2.1916		
6.34	0.7277	6.64	0.8258	6.94	0.9327	8.00	1.3866	9.45	2.2251		
6.35	0.7308	6.65	0.8292	6.95	0.9364	8.05	1.4112	9.50	2.2590		
6.36	0.7340	6.66	0.8326	6.96	0.9401	8.10	1.4360	9.55	2.2932		
6.37	0.7371	6.67	0.8361	6.97	0.9439	8.15	1.4611	9.60	2.3278		
6.38	0.7403	6.68	0.8395	6.98	0.9476	8.20	1.4866	9.65	2.3627		
6.39	0.7435	6.69	0.8430	6.99	0.9514	8.25	1.5123	9.70	2.3980		

8.2.1 Young's modulus can be expressed in multiples of Pa (preferably GPa) after the values have been calculated in 8.2 in psi.

8.3 If it is desired to make all measurements, calculations, and corrections in metric or SI units, reference may be made to related sections of Test Methods C 623, C 848, and C 747 for SI units. (Test Method C 215 uses U.S. customary units, as is done in 8.1 and 8.2.)

9. Report

9.1 Report the following information:

9.1.1 All measurements necessary to calculate Young's modulus for all specimens tested,

9.1.2 Young's modulus (modulus of elasticity) for each specimen tested, to three digits, and

9.1.3 Average Young's modulus (modulus of elasticity) for all specimens tested of a sample lot.

TABLE 2 Nominal Sizes of Test Bars

	Dimensions, in. (mm)	Weight, lb (g)
Fused silica	4 by 13⁄16 by 5⁄32 (100 by 20 by 4)	0.04 (18)
	4 by 13⁄16 by 11⁄32 (100 by 20 by 9)	0.10 (45)
	4 by 13⁄16 by 3⁄4 (100 by 20 by 19)	0.20 (91)
Aluminum	6 by 2 by 1 (150 by 50 by 25)	1.16 (526)

10. Precision and Bias [6]

10.1 Data—An interlaboratory study was initiated in 1977 with eight laboratories using test bars cut from fused silica and aluminum. Three thicknesses of fused silica bars were used to test approximate resonance frequency levels of 2, 5, and 10 kHz. The aluminum bars were sized to achieve approximately 5 kHz resonance frequency.

10.1.1 The nominal sizes of the test bars were as described in Table 2.

10.1.2 All laboratories tested the same specimens, but not all laboratories succeeded in testing the fused silica bars successfully because of their small size. It is important to note that heavy-duty test equipment cannot meet the criteria under Section 7 (especially Note 4) regarding small specimens.

10.1.3 Five laboratories completed testing five specimens each of the fused silica, and six laboratories tested thirteen specimens of the aluminum.

10.2 *Precision*:

10.2.1 Precision is based on the measurement of resonance frequency only. For averages of the specimens tested within one laboratory, their difference is considered significant for a probability of 95 % and t = 1.96, if it equals or exceeds the repeatability intervals listed for precision in Table 3 or for relative precision in Table 4. Likewise, the difference between averages obtained by two laboratories is considered significant if it equals or exceeds the applicable reproducibility intervals in Table 3 and Table 4.

10.2.2 The user is cautioned that precision and relative precision both decrease as specimen size and mass decreases.

10.3 *Bias*—No information can be presented on the bias of the procedure in Test Method C 885 for measuring Young's Modulus because no material having an accepted reference value is available.

11. Keywords

11.1 flexural vibration; monolithic refractories; refractory brick; sonic resonance; Young's Modulus

[6] Supporting data are available from ASTM Headquarters. Request RR: C 08-1005.

TABLE 3 Precision[A]

NOTE—The sample size of the materials may be found in Table 2.

Material	Average $/AmX_j$, Hz	Standard Deviations		Repeatability Interval		Reproducibility Interval	
		Within Laboratories s(W), Hz	Between Laboratories s(L), Hz	m = 1[A], Hz	m = 5, Hz	m = 1, Hz	m = 5, Hz
Fused silica	2245	69	68	191	85	268	207
Fused silica	5175	71	107	197	88	356	309
Fused silica	9854	26	9	72	32	76	41
Aluminum	5279	22	21	62	28	85	64

[A] m = number of replicates, 95 % probability, t = 1.96.

TABLE 4 Relative Precision[A]

NOTE— The sample size of the materials may be found in Table 2.

Material	Average $/AmX_j$, Hz	Coefficients of Variation		Repeatability Interval		Reproducibility Interval	
		Within Laboratories CV(W), %	Between Laboratories CV(L), %	m = 1[A], %	m = 5, %	m = 1, %	m = 5, %
Fused silica	2245	3.07	3.03	8.51	3.79	11.94	9.22
Fused silica	5175	1.37	2.07	3.81	1.70	6.88	5.97
Fused silica	9854	0.26	0.01	0.73	0.32	0.77	0.42
Aluminum	5279	0.42	0.39	1.18	0.52	1.60	1.21

[A] m = number of replicates, 95 % probability, t = 1.96.

APPENDIX

(Nonmandatory Information)

X1. METHOD OF CALCULATING SHAPE CONSTANT [C_1b]

X1.1 The constant [C_1b] depends upon the shape and size of specimens, the mode of vibration, and Poisson's ratio.

X1.2 Using Pickett's equations[5] for rectangular prisms, a Poisson's ratio of ⅙ , and the first mode of vibration in flexure, [C_1b] (in s^2/in.) is determined as follows:

$$[C_1b] = 0.002452(l/t)^3 T_1 \tag{X1.1}$$

where T_1 is a correction factor.

X1.3 For a Poisson's ratio of ⅙ and reciprocal slenderness ratios (r/l) up to 0.3, the following equation holds for T_1:

$$T_1 = 1 + 81.79(r/l)^2 - \frac{1314(r/l)^4}{1 + 81.09(r/l)^2} - 125(r/l)^4 \tag{X1.2}$$

where r is the radius of gyration, which for a rectangular prism equals 0.289 t. Combining Eq X1.1 and Eq X1.2, [C_1b] will be as follows:

$$[C_1b] = 0.002452(l/t)^3 \times \left[1 + 6.8312(t/l)^2 - \frac{9.1661(t/l)^4}{1 + 6.7727(t/l)^2} - 0.8720(t/l)^4\right] \tag{X1.3}$$

Young's modulus E is then calculated as follows:

$$E\text{, psi} = \frac{[C_1b]}{b} wf^2 \tag{X1.4}$$

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