Standard Guide for Dynamic Testing of Vulcanized Rubber and Rubber-Like Materials Using Vibratory Methods[1]

This standard is issued under the fixed designation D 5992; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

ϵ^1 NOTE—Editorial changes were made throughout in July 2001.

1. Scope

1.1 This guide covers dynamic testing of vulcanized rubber and rubber-like (both hereinafter termed "rubber" or "elastomeric") materials and products, leading from the definitions of terms used, through the basic mathematics and symbols, to the measurement of stiffness and damping, and finally through the use of specimen geometry and flexing method, to the measurement of dynamic modulus.

1.2 This guide describes a variety of vibratory methods for determining dynamic properties, presenting them as options, not as requirements. The methods involve free resonant vibration, and forced resonant and nonresonant vibration. In the latter two cases the input is assumed to be sinusoidal.

1.3 While the methods are primarily for the measurement of modulus, a material property, they may in many cases be applied to measurements of the properties of full-scale products.

1.4 The methods described are primarily useful over the range of temperatures from −70°C to +200°C (−100°F to +400°F) and for frequencies from 0.01 to 100 Hz. Not all instruments and methods will accommodate the entire ranges.

1.5 When employed for the measurement of dynamic modulus, the methods are intended for materials having complex moduli in the range from 100 to 100 000 kPa (15 to 15 000 psi) and damping angles from 0 to 90 degrees. Not all instruments and methods will accommodate the entire ranges.

1.6 Both translational and rotational methods are described. To simplify generic descriptions, the terminology of translation is used. The subject matter applies equally to the rotational mode, substituting "torque" and "angular deflection" for "force" and "displacement."

1.7 The values stated in SI units are to be regarded as the standard. The values given in parentheses are for information only.

1.8 This guide is divided into sections, some of which include:

1.9 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

D 945 Test Methods for Rubber Properties in Compression or Shear (Mechanical Oscillograph)[2]

D 1566 Terminology Relating to Rubber[2]

2.2 *ISO Document:*[3]

ISO 2856 Elastomers—General Requirements for Dynamic Testing

2.3 *DIN Document:*[4]

DIN 53 513 Determination of viscoelastic properties of

[1] This guide is under the jurisdiction of ASTM Committee D11 on Rubber and is the direct responsibility of Subcommittee D11.14 on Time and Temperature-Dependent Physical Properties.

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[2] *Annual Book of ASTM Standards*, Vol 09.01.

[3] Available from American National Standards Institute, 25 W. 43rd St., 4th Floor, New York, NY 10036.

[4] Available from Deutsches Institut fur Normung, Burggrafenstr 4-7, 1 Berlin 30, Germany.

elastomers on exposure to forced vibration at non-resonant frequencies

3. Terminology

3.1 Definitions—The following terms are listed in related groups rather than alphabetically (see also Terminology D 1566).

3.1.1 *delta,* δ, *n— in the measurement of rubber properties*, the symbol for the phase angle by which the dynamic force leads the dynamic deflection; mathematically true only when the two dynamic waveforms are sine waves (Synonym— *loss angle*).

3.1.2 *tandel, tan*δ, *n*—mathematical tangent of the phase angle delta (δ); pure numeric; often written spaced: tan del; often written using "delta": tandelta, tan delta (Synonym—*loss factor*).

3.1.3 *phase angle*, *n*—in general, the angle by which one sine wave leads another; units are either radians or degrees.

3.1.4 *loss angle*, *n*—synonym for delta (δ).

3.1.5 *loss factor*, *n*—synonym for tandel (tanδ) (η).

3.1.6 *damping*, *n*—that property of a material or system that causes it to convert mechanical energy to heat when subjected to deflection; in rubber the property is caused by hysteresis; in some types of systems it is caused by friction or viscous behavior.

3.1.7 *hysteresis*, *n*—the phenomenon taking place within rubber undergoing strain that causes conversion of mechanical energy to heat, and which, in the "rubbery" region of behavior (as distinct from the glassy or transition regions), produces forces essentially independent of frequency. (See also *hysteretic* and *viscous*.)

3.1.8 *hysteresis loss*, *n*—per cycle, the amount of mechanical energy converted to heat due to straining; mathematically, the area within the hysteresis loop, having units of the product of force and length.

3.1.9 *hysteresis loop*, *n*—the Lissajous figure, or closed curve, formed by plotting dynamic force against dynamic deflection for a complete cycle.

3.1.10 *hysteretic*, *adj— as a modifier of damping*, descriptive of that type of damping in which the damping force is proportional to the amplitude of motion across the damping element.

3.1.11 *viscous*, *adj—as a modifier of damping*, descriptive of that type of damping in which the damping force is proportional to the velocity of motion across the damping element, so named because of its derivation from an oil-filled dashpot damper.

3.1.12 *equivalent viscous damping, c*, *n*— at a given frequency, the quotient of F″(1) divided by the velocity of the imposed deflection.

$$c = F''(1) / \omega X^*(1) \quad (1)$$

3.1.12.1 *Discussion*—The equivalent viscous damping is useful when dealing with equations in many texts on vibration. It is an equivalent only at the frequency for which it is calculated.

3.1.13 *dynamic*, *adj—in testing*, descriptive of a force or deflection function characterized by an oscillatory or transient condition, as contrasted to a static test.

3.1.14 *dynamic*, *adj—as a modifier of stiffness or modulus*, descriptive of the property measured in a test employing an oscillatory force or motion, usually sinusoidal.

3.1.15 *static*, *adj (1)—in testing*, descriptive of a test in which force or deflection is caused to change at a slow constant rate, within or in imitation of tests performed in screw-operated universal test machines.

3.1.16 *static*, *adj (2)—in testing*, descriptive of a test in which force or deflection is applied and then is truly unchanging over the duration of the test, often as the mean value of a dynamic test condition.

3.1.17 *static*, *adj (3)—as a modifier of stiffness or modulus*, descriptive of the property measured in a test performed at a slow constant rate.

3.1.18 *stiffness*, *n*—that property of a specimen that determines the force with which it resists deflection, or the deflection with which it responds to an applied force; may be static or dynamic (See also *complex, elastic, damping*.) (Synonym—*spring rate*).

3.1.19 *modulus*, *n*—the ratio of stress to strain; that property of a material which, together with the geometry of a specimen, determines the stiffness of the specimen; may be static or dynamic, and if dynamic, is mathematically a vector quantity, the phase of which is determined by the phase of the complex force relative to that of deflection. (See also *complex*, *elastic*, *damping*.)

3.1.20 *complex*, *adj—as a modifier of dynamic force*, descriptive of the total force; denoted by the asterisk (*) as a superscript symbol (F*); F* can be resolved into elastic and damping components using the phase of displacement as reference.

3.1.21 *elastic*, *adj—as a modifier of dynamic force*, descriptive of that component of complex force in phase with dynamic deflection, that does not convert mechanical energy to heat, and that can return energy to an oscillating mass-spring system; denoted by the single prime (′) as a superscript symbol, as F′.

3.1.22 *damping*, *adj—as a modifier of dynamic force*, descriptive of that component of complex force leading dynamic deflection by 90 degrees, and that is responsible for the conversion of mechanical energy to heat; denoted by the double prime (″) as a superscript symbol, as F″.

3.1.23 *storage*, *adj—as a modifier of energy*, descriptive of that component of energy absorbed by a strained elastomer that is not converted to heat and is available for return to the overall mechanical system; by extension, descriptive of that component of modulus or stiffness that is elastic.

3.1.24 *Fourier analysis*, *n— in mathematics*, analysis of a periodic time varying function that produces an infinite series of sines and cosines consisting of a fundamental and integer harmonics which, if added together, would recreate the original function; named after the French mathematician Joseph Fourier, 1768–1830.

3.1.25 *shear*, *adj*—descriptive of properties measured using a specimen deformed in shear, for example, shear modulus.

3.1.26 *bonded*, *adj—in describing a test specimen*, one in which the elastomer to be tested is permanently cemented to members of much higher modulus for two purposes: (*1*) to provide convenient rigid attachment to the test machine, and (*2*) to define known areas for the application of forces to the elastomer.

3.1.27 *unbonded*, *adj—in describing a test specimen*, one in which the elastomer is molded or cut to shape, but that otherwise demands that forces be applied directly to the elastomer.

3.1.28 *bond area*, *n—in describing a bonded test specimen*, the cemented area between elastomer and high-modulus attachment member.

3.1.29 *contact area*, *n— in an unbonded specimen*, that area in contact with a high-modulus fixture, and through which applied forces pass; may or may not be constant, and if lubricated, may deliberately be allowed to change.

3.1.30 *lubricated*, *adj— in describing an elastomeric test specimen having at least two plane parallel faces and to be tested in compression*, one in which the plane parallel faces are separated from plane parallel platens of the apparatus by a lubricant, thereby eliminating, insofar as possible, friction between the elastomer and platens, permitting the contact surfaces of the specimen to expand in area as the platens are moved closer together.

3.1.31 *Mullins Effect*, *n*—the phenomenon occurring in vulcanized rubber whereby the second and succeeding hysteresis loops exhibit less area than the first, due to breaking of physical cross-links; may be permanent or temporary, depending on the nature of the material. (See also *preflex effect*.)

3.1.32 *preflex effect*, *n*—the phenomenon occurring in vulcanized rubber, related to the Mullins effect, whereby the dynamic moduli at low strain amplitude are less after a history to high strains than before. (See also *Mullins effect*.) (Also called strain history effect.)

3.1.33 *undamped natural frequency*, *n—in a single-degree-of-freedom resonant spring-mass-damper system*, that natural frequency calculated using the following equation:

$$f_n = \mathrm{SQRT}\ (K'/M) \quad (2)$$

where:

K' = the elastic stiffness of the spring, and
M = the mass.

3.1.34 *transmissibility*, *n— in the measurement of forced resonant vibration*, the complex quotient of response divided by input; may be absolute or relative.

3.1.35 *absolute*, *adj—in the measurement of vibration*, a quantity measured relative to the earth as reference.

3.1.36 *relative*, *adj—in the measurement of vibration*, a quantity measured relative to another body as reference.

3.1.37 *LVDT*, *n*—abbreviation for ′′Linear Variable Differential Transformer,” a type of displacement transducer characterized by having a primary and two secondary coils arranged along a common axis, the primary being in the center, and a movable magnetic core free to move along the axis that induces a signal proportional to the distance from the center of its travel, and of a polarity determined by the phase of the signals from the two secondary coils. The rotary version is called a Rotary Variable Differential Transformer (RVDT).

3.1.38 *mobility analysis*, *n*—the science of analysis of mechanical systems employing a vector quantity called “mobility,” characteristic of lumped constant mechanical elements (mass, stiffness, damping), and equal in magnitude to the force through the element divided by the velocity across the element.

3.1.39 *impedance analysis*, *n*—the science of analysis of mechanical systems employing a vector quantity called “impedance,” characteristic of lumped constant mechanical elements (mass, stiffness, damping), and equal in magnitude to the velocity across the element divided by the force through the element.

3.1.39.1 *Discussion*—Mobility analysis is sometimes easier to employ than impedance because mechanical circuit diagrams are more intuitively constructed in the mobility system. Either will provide the understanding necessary in analyzing test apparatus.

3.2 *Symbols:Symbols:*

3.2.1 *General Comments*:

3.2.1.1 Many symbols use parentheses. The (t) denotes a function of time. When enclosing a number, such as (1) or (2), the reference is to the number or “order” of the harmonic obtained through Fourier analysis (see Appendix X2). Thus, all of the parameters indicated as (1) are obtained from the fundamental, or first, harmonic. A second harmonic from the complex force would be denoted as F*(2), etc. It should be noted that each harmonic has a phase angle associated with it. In the case of the fundamental, it is the loss angle (δ). The phase angles become important for the higher harmonics if the reverse Fourier transform is employed to reconstitute data in the time domain.

3.2.1.2 Three superscripts are used: the asterisk (*), the single prime (′), and the double prime (″). This guide discusses dynamic motion and force. As raw data, each is a “complex” parameter, denoted by the asterisk. In this guide force is referenced to motion for its phase. The component of force in phase with motion is denoted by a single prime; the component leading motion by 90 degrees is denoted by the double prime. Quantities deriving from force, such as stress, stiffness, and modulus, like force, are also vector quantities and use the same superscripts to identify their phase relationship.

3.2.1.3 In some literature, the asterisk is omitted from the parameter imposed on the specimen. Thus X*(t) is often abbreviated X(t) for a motion-excited system, F*(t) as F(t) in a force-excited one. This guide uses the longer complete form for both.

3.2.2 *Motion, Force and Stiffness*:

3.2.2.1 Following are definitions of symbols describing test parameters and quantities derived from them, presented in the order in which they become available and are used. In forced nonresonant apparatus, X*(t) and F*(t) are measured directly by deflection and force transducers.

$X^*(t)$ = dynamic deflection of the specimen as a function of time.
$F^*(t)$ = dynamic complex force as a function of time.
$X^*(1)$ = dynamic deflection, single amplitude, of the fundamental component of X*(t); obtained by Fourier analysis or equivalent means.

$F^*(1)$ = dynamic complex force, single amplitude, of the fundamental component of F*(t), obtained by Fourier analysis or equivalent means.
δ = phase angle by which F*(1) leads X*(1); only true of F*(t) and X*(t) if both are pure sine waves, which does not occur with most elastomers.
η = $\tan\delta = F''(1)/F'(1)$ = loss factor.
$F'(1)$ = $F^*(1)\cos\delta$ = dynamic elastic force, single amplitude; that component of F*(1) in phase with X*(1).
$F''(1)$ = $F^*(1)\sin\delta$ = dynamic damping force, single amplitude; that component of F*(1) leading X*(1) by 90 degrees.
$K^*(1)$ = F*(1)/X*(1) = dynamic complex stiffness, magnitude, obtained by taking the ratio of F*(1) and X*(1); has the phase of F*(1).
$K'(1)$ = F′(1)/X*(1) = dynamic elastic stiffness, magnitude, obtained by taking the ratio of F′(1) and X*(1); has the phase of F′(1).
$K''(1)$ = $F''(1)/X^*(1)$ = dynamic damping stiffness, magnitude, obtained by taking the ratio of F″(1) and X*(1); has the phase of F″(1).

3.2.2.2 From the last three, the dynamic stiffnesses, three corresponding dynamic moduli can be calculated using geometric factors appropriate to the specimen. In the case of shear moduli, the symbols are G*(1), G′(1), and G″(1). For extension or compression moduli, the symbols are E*(1), E′(1), and E″(1). Appendixes X2, X3, and X4 give the relationships for three common geometries.

3.2.3 *Resonant Systems*:

3.2.3.1 Additional symbols are used with resonant systems to describe the imposed and response motions and forces:

ϕ = phase angle by which either the imposed force or base motion leads the motion of the mass. (Should not be confused with the phase angle δ which is the angle by which complex force through a specimen leads the deflection across the specimen.)
β = frequency ratio, the quotient of the frequency of interest divided by the undamped natural frequency.
ζ = viscous damping ratio, c/c_c.
ω_n = undamped natural frequency, radians per second.
f_n = undamped natural frequency, Hz.

3.2.4 *Symbols for Torsion*:

3.2.4.1 Torsion functions are analogous to those of translation. The corresponding symbols and units are:

	Translational	Torsional
Displacement	X	θ
Units, SI	mm	radian
Units, English	inch	radian
Force, torque	F	S
Units, SI	newton	newton metre
Units, English	pound	pound inch

The asterisk, single and double prime, and parentheses are used exactly as for translational cases.

3.2.5 *Voltage Symbols*:

3.2.5.1 Symbols used to describe voltage signals from instrumentation require subscripts to identify what they represent. Hence, for example, E_x represents a voltage proportional to motion, and E_F a voltage proportional to force. Here also the asterisk, single and double primes, and the parentheses are used as with their corresponding mechanical counterparts.

3.2.6 *Geometric Symbols*:

3.2.6.1 Symbols used to describe specimens and apparatus are defined in the figures depicting the methods and specimens involved. A few symbols have been preempted. For instance, t always indicates time, never thickness. Frequency, not force, preempts the lower case *f*; force must use the upper case *F*. Dimensional symbols such as *a*, *b*, *r*, and *L* will have assignments specific to a particular specimen geometry and will mean other things in other geometries.

4. Summary

4.1 The methods covered in this guide are divided into three general categories:

4.1.1 Forced nonresonant vibration,

4.1.2 Free resonant vibration, and

4.1.3 Forced resonant vibration.

4.2 Brief descriptions of representative methods in each category are given, together with sufficient mathematical formulae to indicate how results are calculated and presented.

5. Significance and Use

5.1 This guide is intended to describe various methods for determining the dynamic properties of vulcanized rubber materials, and by extension, products utilizing such materials in applications such as springs, dampers, and flexible load-carrying devices, flexible power transmission couplings, vibration isolation components and mechanical rubber goods in general. As a guide, it is intended to provide descriptions of options available rather than to specify the use of any one in particular.

6. Hazards

6.1 There are no hazards inherent in the methods described; there are no reagents or hazardous materials used. The machinery used may be potentially hazardous, especially in forced nonresonant testing machines. These may involve the creation of significant forces and motions, and may move unexpectedly. Caution should always be used when operating such machinery. The problem is especially acute in servohydraulic machinery, which is at once the most versatile yet potentially dangerous class of machines used in dynamic testing. The design of machines and fixtures should be done with this in mind; pinch points should be eliminated or guarded.

6.2 Normal safety precautions and good laboratory practice should be observed when setting up and operating any equipment. This is especially true when tests are to be performed at low or high temperatures, when flammable coolants or electrical heaters are apt to be used.

7. Factors Influencing Dynamic Measurement

7.1 Dynamic measurement of rubber is influenced by three major factors: (*1*) thermodynamic, having to do with the internal temperature of the specimen; (*2*) mechanical, having to do with the test apparatus; and (*3*) instrumentation and electronics, having to do with the ability to obtain and handle signals proportional to the needed physical parameters. The latter two factors are discussed in detail in Annex A1. The thermodynamic factor will be examined in 7.2 and 7.3.

7.2 Any rubber specimen exhibits a rise in internal temperature with mechanical strain. The magnitude of the rise is dependent on the damping coefficient (tandel), and the thermal properties and geometry of the rubber and metal. It is axiomatic that the thermodynamic behavior of a laboratory modulus-measurement specimen is never exactly like that of the commercial product whose behavior is to be predicted. Accordingly, the laboratory engineer and the product designer must work together. The laboratory must produce elastic and damping data for the rubber, measured with the entire body of rubber at the temperature reported. This needs to be done over a range of temperatures, frequencies, and strains, selected after consultation with the product designer. The designer then must take this information and predict the internal temperature of the product. This will require knowing the geometry and thermal properties of the rubber, the heat sink or source ability of attached metal or other rigid parts, the service conditions of motion, frequency, initial temperature, and operating time. Prediction may be an iterative process, where the first calculated temperature changes the stiffness and damping, which change the strain, which in turn changes the heat dissipation and hence the temperature, etc.

7.3 To put this matter in perspective, rubber having a loss factor of 0.7 may rise as much as 0.5°C (1°F) for each cycle of motion if the shear strain amplitude is ±100 percent. Lesser strains, and lesser values of tandel, produce less temperature rise. The possibility of significant temperature rise, relative to the reported ambient or starting temperatures, points out the desirability of methods capable of performing a dynamic test in as few cycles as possible. Where many test conditions must be imposed, it is necessary to pause an appropriate time between conditions for the temperature to return to its specified value. Good heat sinking, either by conduction from the rubber to the grips, or by forced convection, helps in maintaining the desired temperature.

7.4 The selection of test apparatus and test method are influenced by the material discussed in 7.2 and 7.3. Especially in the measurement of modulus, a balance must be found between the needs of the analysis equipment for data (can it acquire data in a few cycles), the need of the modulus measurement to be at a known temperature (few cycles), and the probable need of the elastomer to be past the experience of the Mullins Effect (past the first cycle). Conversely, a stiffness measurement on a full-scale product might be desired at either a known temperature (few cycles) or at steady state, the latter requiring a heat sink typical of service.

8. Test Methods and Specimens

8.1 *Introduction*:

8.1.1 Three basic vibratory methods exist:

8.1.1.1 Forced vibration of a nonresonant system involving only the specimen,

8.1.1.2 Free vibration of a resonant system involving the specimen and a mass, and

8.1.1.3 Forced vibration of the above resonant system.

8.1.2 The first and third can be broken down further into two kinds of apparatus, those that impose a dynamic motion and those that impose a dynamic force. The imposed parameter could have any of the following wave shapes: sinusoidal, triangular, square, or random. In this guide we will assume that the imposed parameter is always sinusoidal.

8.1.3 In addition to the availability of three methods, there is also a choice of specimen geometry. Elastomers may be strained in:

8.1.3.1 *Shear*—May be single, double, or quad. Usually double, with two identical rubber elements symmetrically disposed on opposite sides of a central rigid member.

8.1.3.2 *Compression*—May be bonded, unbonded, or lubricated.

8.1.3.3 *Tension*.

8.1.3.4 *Bending*—May be fixed-free, fixed-fixed, fixed-guided, or three-point bending of beams.

8.1.3.5 *Torsion*.

8.1.4 Some materials exhibit a large change in dynamic modulus with change in dynamic strain. In applications where this is important, attention should be paid to whether the specimen geometry and flexing method impose uniform strain throughout the body of the specimen, or whether the strain varies within the specimen.

8.2 *Forced Nonresonant Vibration*:

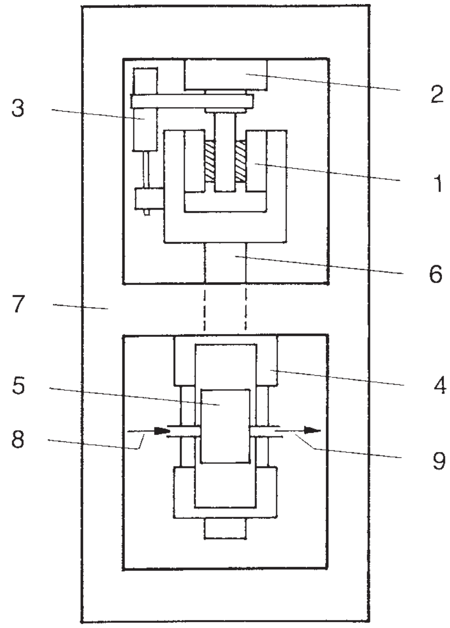
8.2.1 Forced nonresonant vibration offers the broadest frequency range of all methods. It can be accomplished with mechanical crank-and-link mechanisms, with electrodynamic linear force motors, and with servohydraulics. When done with electrodynamics or servohydraulics it adds ease of amplitude adjustment. Servohydraulics offers, as well, the possibility of obtaining the required data in as few as one cycle, which makes temperature rise during the test negligible.

8.2.2 A typical servohydraulic test system is depicted in Fig. 1. An all-mechanical system having many of the same features is shown in Fig. 2. The former has the possibility of imposing either motion or force as the input. The mechanical system shown can apply only vibratory motion. Fig. 3 shows an electrodynamically excited system. (All-mechanical machines using rotating weights or oscillating masses to develop sinusoidal forces are possible but are extremely complex, and will not be dealt with here.)

8.3 *Free Resonant Vibration*:

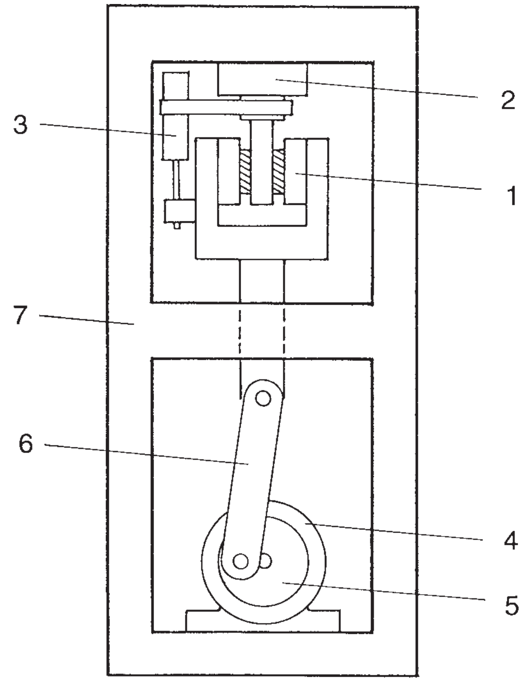
8.3.1 Any resonant system consists of two essential elements: a spring and a mass. A third element, a damper, may be added to cause decay of the resonant vibration amplitude. In elastomers, the elasticity (springiness) and damping are both inherent in the material. Testing by free resonant vibration involves deflecting the specimen, then releasing it and allowing the mass to oscillate freely (hence "free" vibration) at a frequency determined by the stiffness of the specimen and the magnitude of the mass. This frequency of natural oscillation is termed, appropriately, the "natural frequency."

8.3.2 As the mass and spring oscillate, they pass energy back and forth. It alternately takes the form of stored and kinetic energy. Some is lost to damping and is converted to heat. As it is lost, the oscillatory amplitude becomes less and less, or "decays." By measuring the deflection amplitude of each successive cycle, a measure of damping can be had through the application of the logarithmic decrement, or "log



1 Test Specimen
2 Force Transducer
3 Deflection Transducer
4 Hydraulic Actuator
5 Electrohydraulic Servovalve
6 Actuator Piston Rod
7 Rigid Frame
8 Hydraulic Supply
9 Hydraulic Return

FIG. 1 Typical Servohydraulic Test System



1 Test Specimen
2 Force Transducer
3 Deflection Transducer
4 Motor
5 Eccentric (may be adjustable)
6 Connecting Rod
7 Rigid Frame

FIG. 2 Typical All-Mechanical Test System

decrement," for which the symbol is Δ. Fig. 4 illustrates how the peaks of vibratory response decay with time.

8.3.3 This method has the advantage of requiring little equipment, but suffers the inherent and serious problem of not being able to provide a constant strain amplitude. This poses a problem in determining the influence of dynamic strain on elastic and damping stiffnesses. With highly damped elastomers the technique is difficult to apply because so few cycles are available for use. The equations for logarithmic decrement in terms of the decaying amplitudes assume linearity and that moduli are not influenced by strain amplitude.

8.3.4 The same decay curve from which log decrement is obtained can be used to calculate specimen stiffness. Calculation of log decrement utilized the amplitudes; calculation of stiffness will use the period of oscillation and knowledge of the mass if translational, or of the moment of inertia if torsional.

8.3.5 Appendix X5 explains the method in more detail and gives the equations for log decrement, loss factor, and stiffness.

8.4 *Forced Resonant Vibration*:

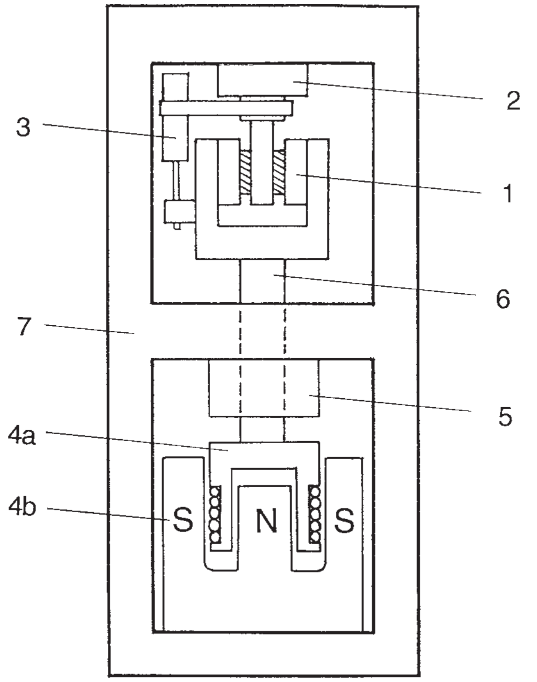
8.4.1 As with the free resonant system, the elastomeric spring with its inherent damping, and a mass, are necessary. This method, however, requires an external source of vibratory energy. Two sources are possible: motion excitation ("shake table") and force excitation. The shake table case is the easier of the two to implement, and is the only one described.

8.4.2 Traditional texts on vibration theory deal with systems using purely elastic springs, and viscous dampers. It is important to note the difference between viscous damping and that which occurs in rubber. Force due to viscous damping is proportional to velocity, and hence is a first-power function of frequency; the damping force in elastomers is nearly independent of frequency. To use most textbook equations with elastomeric isolators one must use an "equivalent viscous damping" for the particular frequency of interest, or use an entirely different model. The model based on "hysteretic damping" is a better representation of the damping behavior of typical elastomers. This model is also often referred to as "complex," "solid," or "structural" damping.

8.4.3 *Absolute and Relative Motions*:

8.4.3.1 In considering forced resonant vibration it is important to distinguish between "absolute" and "relative" transmissibility and phase. Absolute transmissibility is the amplitude of the response motion of the supported mass divided by the amplitude of the input motion. Absolute phase is the phase angle between the above two quantities, considered as sine waves. Relative transmissibility is the amplitude of deflection of the elastomeric spring divided by the amplitude of motion of the shake table input. Relative phase is the phase angle between these two quantities, considered as sine waves. The response motion of the supported mass and the deflection of the specimen are entirely different things. Not all texts explain these relationships clearly.

8.4.3.2 Figs. 5-7 show the case of absolute transmissibility and phase. Figs. 8-10 show relative transmissibility and phase. The plots shown are theoretical curves based on mathematical equations; they are not test data. In all of them, "frequency ratio" β is the ratio of the vibration frequency to the undamped natural frequency. Undamped natural frequency is calculated using the mass and the elastic stiffness, using Eq X5.5 or Eq



1	Test Specimen	4b	Actuator Magnet
2	Force Transducer	5	Piston Guide Bearing
3	Deflection Transducer	6	Piston Rod
4a	Actuator Coil	7	Rigid Frame

FIG. 3 Typical Test System Utilizing an Electrodynamic Exciter

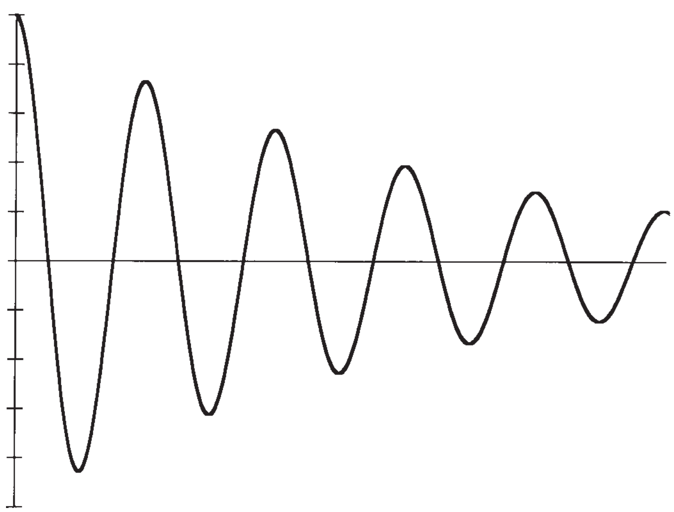


FIG. 4 Typical Decay Wave

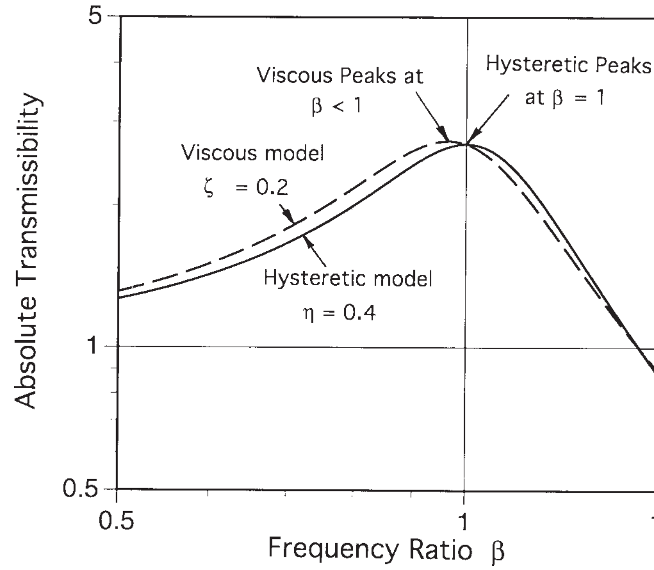


FIG. 5 Absolute Transmissibility Versus β

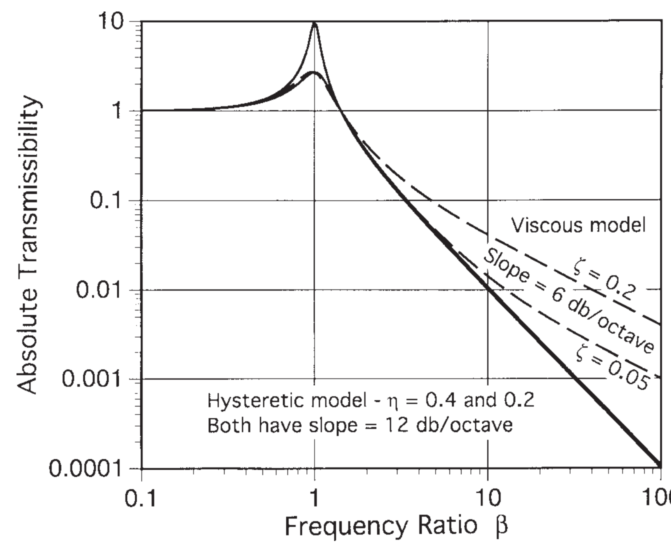


FIG. 6 Absolute Transmissibility Versus β

X5.7 (see Appendix X5). On the plots, the undamped natural frequency is denoted by β = 1.

8.4.4 *Hysteretic and Viscous Damping Effects—Absolute Case*:

8.4.4.1 The curves for absolute transmissibility and phase are not the same for the viscous and hysteretically damped cases. They differ in two ways: (*1*) the frequency at which peak amplitude occurs is different, and (*2*) the slope of the transmissibility curves is different in the isolation range.

8.4.4.2 Fig. 5 and Fig. 6 show absolute transmissibility for both damping types on the same plots for comparison. The first includes only frequencies near peak transmissibility to show clearly how the peaks occur at different frequencies. The second extends well into the isolation range to show how the slopes differ. Viscous damping causes peak absolute transmissibility to occur at β less than unity. With hysteretic damping it always occurs at β = unity. In the range of isolation, the slope of the hysteretically damped case, typical of elastomeric vibration isolators, is 12 dB/octave. The slope of the viscous damped case is 6 dB/octave. Measurement of this slope is one way to demonstrate that elastomers exhibit hysteretic and not viscous damping.

8.4.4.3 Fig. 7 shows absolute phase vs. frequency over the smaller frequency span. In this curve it should be noted that 90 degree phase shift does not occur at β = unity for either viscous or hysteretic damping.

8.4.5 *Hysteretic and Viscous Damping Effects—Relative Case*:

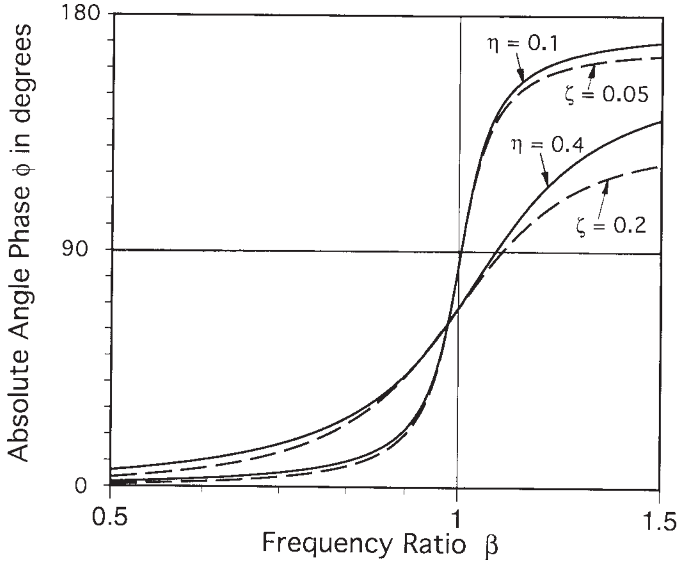


FIG. 7 Absolute Phase Versus β

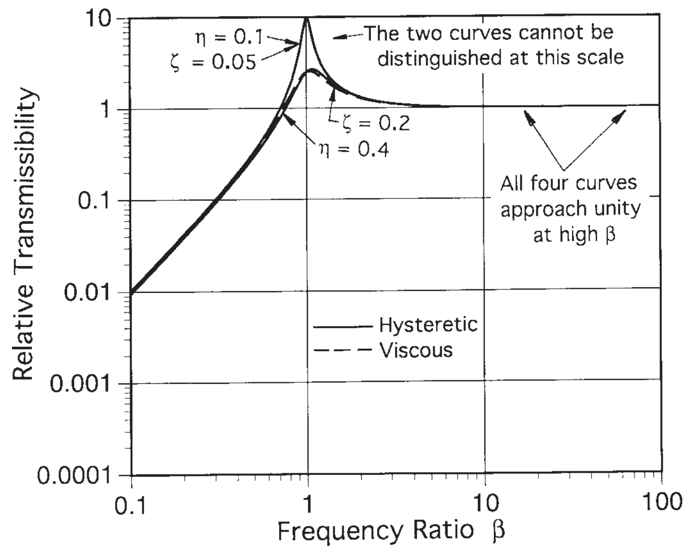


FIG. 9 Relative Transmissibility Versus β

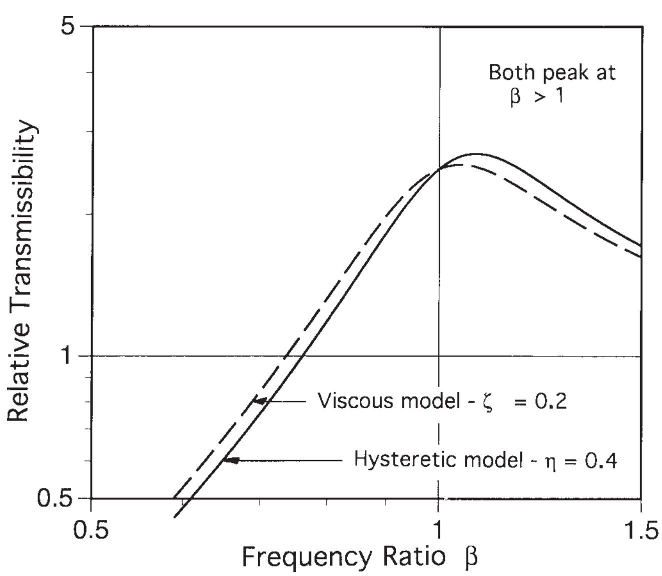


FIG. 8 Relative Transmissibility Versus β

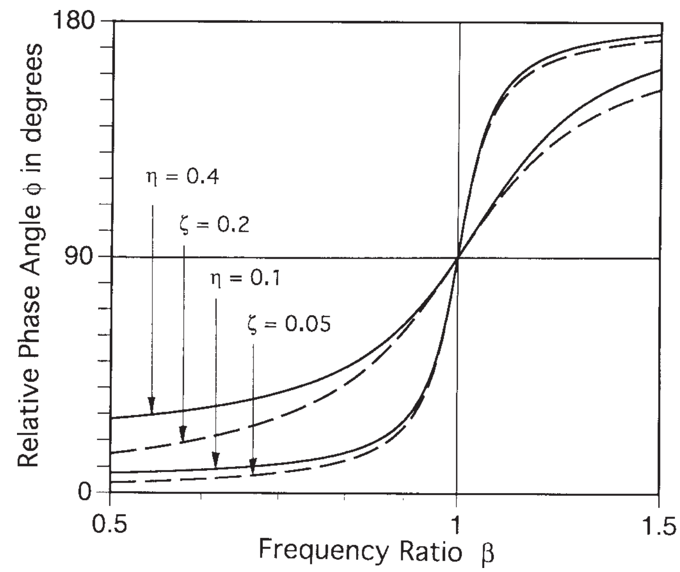


FIG. 10 Relative Phase Versus β

8.4.5.1 Figs. 8-10 examine the same relationships for the case of relative transmissibility and phase. Peak transmissibility now occurs above $\beta = 1$ for both viscous and hysteretic damping. But notice that 90 degree phase shift now occurs at β = unity for both kinds of damping. Fig. 9 shows that both damping models exhibit a uniform transmissibility of unity at high values of β; the displacement of the specimen is equal to the shake table input since the supported mass is isolated; it is stationary.

8.4.5.2 For the shake table relative transmissibility case, for both hysteretic and viscous models, the relative phase angle at the undamped natural frequency is 90 degrees. From the experimenter's standpoint, it would be nice to utilize this fact to determine the undamped natural frequency, and from it the elastic stiffness. Unfortunately, measurement of the dynamic deflection of the specimen (the relative motion) is not easily accomplished.

8.4.6 *Advantages and Disadvantages of Forced Resonant Method*:

8.4.6.1 Determination of dynamic properties over an extended range of frequency is not practical with forced resonant vibration. For a given specimen the only variable available to change the resonant frequency is mass, and it is seldom practical to vary it over a range of more than ten times. This changes the resonant frequency only by a factor of about three (the square root of ten).

8.4.6.2 Compared with free resonant vibration, the forced resonant method has the advantage of allowing the experimenter to adjust for and to maintain a desired resonant amplitude. It has the disadvantage of requiring steady state vibration, and therefore will suffer from internal heat generation within the specimen and consequent change in specimen temperature. See 7 on thermodynamic factors and their influence on dynamic measurement.

8.4.6.3 It should also be noted that the shape of the transmissibility curve will be distorted compared to the theoretical curves by any sensitivity of the elastomer moduli to dynamic strain amplitude and/or frequency. Where this influence is significant, changes in the shape of the transmissibility curve can be expected.

8.4.7 *Obtaining Loss Factor and Stiffness from Forced Resonant Vibration*:

8.4.7.1 Loss factor in general can be determined from the height of the transmissibility curve. Dynamic stiffness in general can be determined from the resonant frequency provided the supported mass (or moment of inertia) is known. Neither relationship is simple if the elastomer has significant damping, for example where tanδ is greater than 0.2. X6.1 gives detailed instructions for obtaining loss factor and dynamic stiffnesses from transmissibility curves, and discusses phase.

8.5 *Choice of Specimen Geometry for Modulus Measurement*:

8.5.1 Thus far, the discussion has been entirely general; the methods described could be used equally well for measurement of dynamic complex stiffness or dynamic complex modulus. Conversion of stiffness results to modulus requires mechanical analysis of the stress and strain in the specimen, since modulus is defined as their quotient. Choosing a specimen geometry depends, therefore, on the degree to which the material is subjected to uniform stress and strain throughout the body of the specimen, and the degree to which this is important for a given measurement.

8.5.2 Elastomers in general are strain sensitive; the dynamic moduli are functions of dynamic strain amplitude. The strength of this relationship varies from compound to compound, becoming more pronounced with increasing stiffness and damping. To the degree that this effect is significant, it implies that the test specimen must be chosen to ensure equal strains at all points within the specimen.

8.5.3 Also implied is that the dynamic strain amplitude must be constant all during the test. This imposes restrictions on the use of free-vibration decay methods, where strain amplitude is constantly changing.

8.6 *Double Shear Specimens*:

8.6.1 For the reasons cited above, the most widely useful specimen for modulus measurement is the double shear type, tested by a forced nonresonant method. Shear, when the geometry is properly selected and height-to-thickness ratio is large (8 to 10), offers near constant strain throughout the specimen.

8.6.2 Fig. 11 illustrates a typical double shear specimen, having its two outer members clamped in a fixture that constrains them to maintain a fixed spacing between them, and to make them move in unison. Depending on the test apparatus, either the outer members or the inner member may be driven by the moving part of the machine, with the other part held stationary.

8.6.3 For double-shear specimens, the relationship between shear modulus of the material, stiffness of the specimen, and geometry of the specimen is derived in Appendix X2. Derivations for tall rectangular, square, and circular cross sections are given, as are the dimensions of recommended specimens.

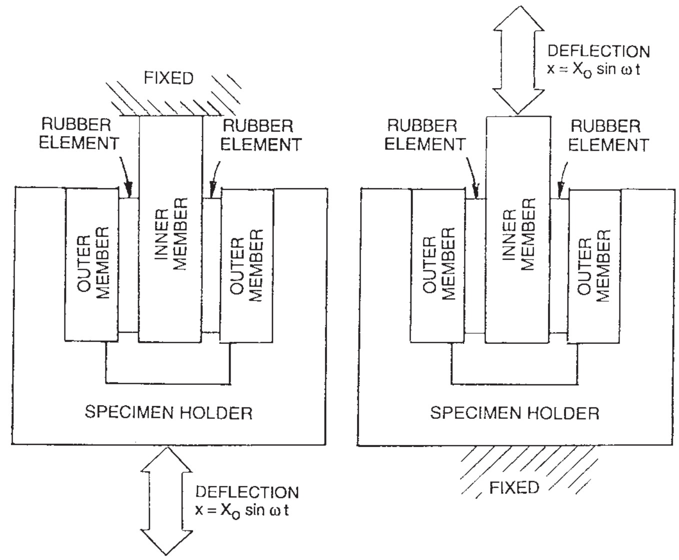


FIG. 11 Double-Shear Specimen, Outer Members Constrained at Constant Spacing

8.7 *Torsion Specimens*:

8.7.1 Fig. 12 shows rectangular and circular cross section specimens twisted in torsion. The formulas for shear modulus and strain are given in Appendix X3. Both figures are for the case of forced nonresonant vibration.

8.8 *Compression/Extension Specimens*:

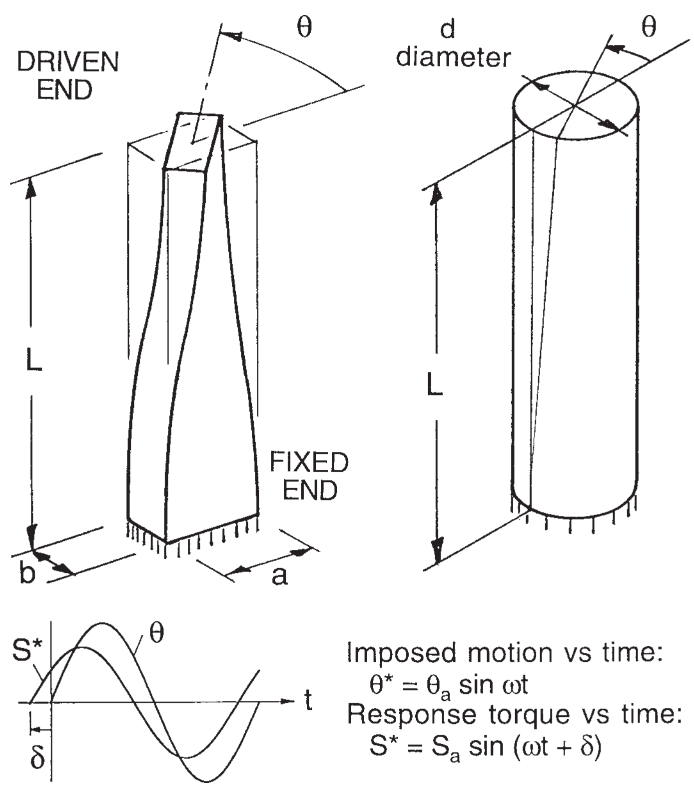


FIG. 12 Torsion Specimens, Rectangular and Circular Cross Sections

8.8.1 Fig. 13 illustrates rectangular and circular cross section specimens loaded in compression and tension (also called extension). The precompressed specimen can be tested as an unbonded button. Both lubricated and nonlubricated methods are used, the latter often with the aid of sandpaper to prevent the contact surface area from changing as the specimen is deflected.

8.8.2 Appendix X4 gives the derivation of equations for extension modulus E as a function of force, deflection, specimen stiffness, and the dimensions of the specimens. Recommended ratios and dimensions, taken from Test Methods D 945, ISO 2856, and DIN 53 513 are given for reference.

8.9 *Bending Specimens*:

8.9.1 Beams in bending are used in several types of apparatus. The types of machines vary in the constraints to which the specimen is subjected. Fig. 14 illustrates diagrammatically some of the constraint schemes in use. Understanding of the end constraints is necessary in order to select the proper analysis equations. In the types shown, a, b, d, and f have the beam length unconstrained; c and e constrain the length to be always at its original length. (The diagrams are schematic; the apparatus may utilize constraints quite different from the roller guides shown.)

8.10 *Tradeoffs Between Methods*:

8.10.1 There are two main considerations in selecting a test method: (*1*) the need for constant strain during the test, and (*2*) heat generation during the test.

8.10.2 Nonresonant, motion excited methods offer constant dynamic input amplitude during the test. Free resonant vibration does not. Of the nonresonant methods, servohydraulics provides the most convenient way to impose a wide variety of test conditions and high forces. (A motion-excited servohydraulic system has its servo loop closed on motion feedback.)

8.10.3 As discussed in 7, if the material has significant damping, it may be desirable to acquire the dynamic data in a burst of a few cycles to minimize temperature rise within the specimen. Nonresonant motion excited methods, especially servohydraulics, are able to do this.

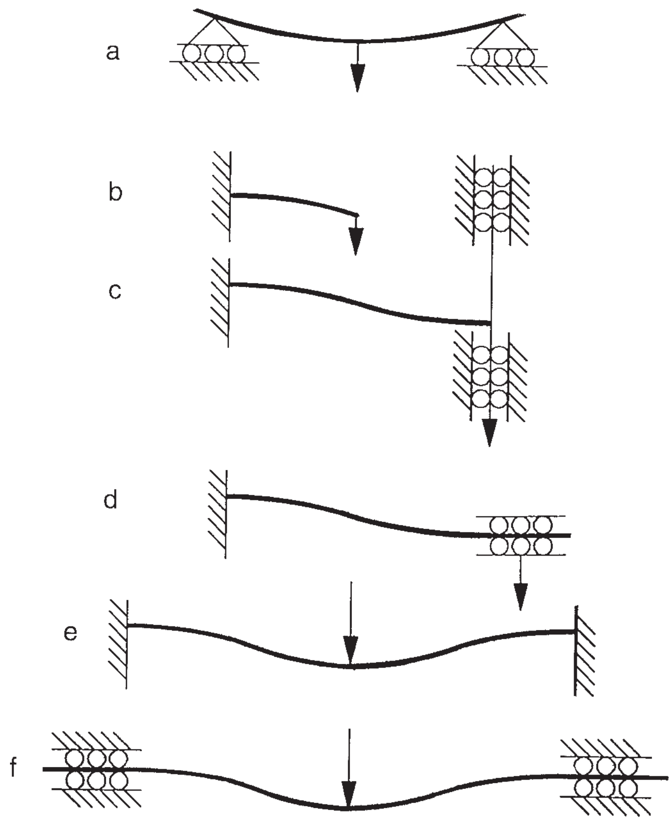


FIG. 14 Bending With Various End Constraints

8.10.4 In general, methods utilizing free resonant vibration are the least expensive, followed by forced resonant methods. Forced nonresonant methods are the most costly, but offer the most comprehensive results.

8.11 *Influences on Accuracy*:

8.11.1 The accuracy of a stiffness measurement can be no better than the accuracy of measurement of force and deflection. Of the two, deflection is usually the more difficult, especially at high frequency where displacements become small. Measurement of force becomes a problem as frequency rises, due to mass reaction forces in fixturing, and the smallness of the forces.

8.11.2 The accuracy of a modulus measurement can be no better than the accuracy of measurement of the dimensions of the specimen. The smallest dimension, usually thickness, is always the most critical. Modulus is calculated from stiffness and specimen geometry; specimen dimensions are critical to accuracy.

8.11.3 The accuracy of a damping measurement can be no better than the excellence of the attachment between specimen and test machine. Grips, fixtures, and the like must not allow slipping, which itself is a form of damping and that, if present, will add to the apparent damping.

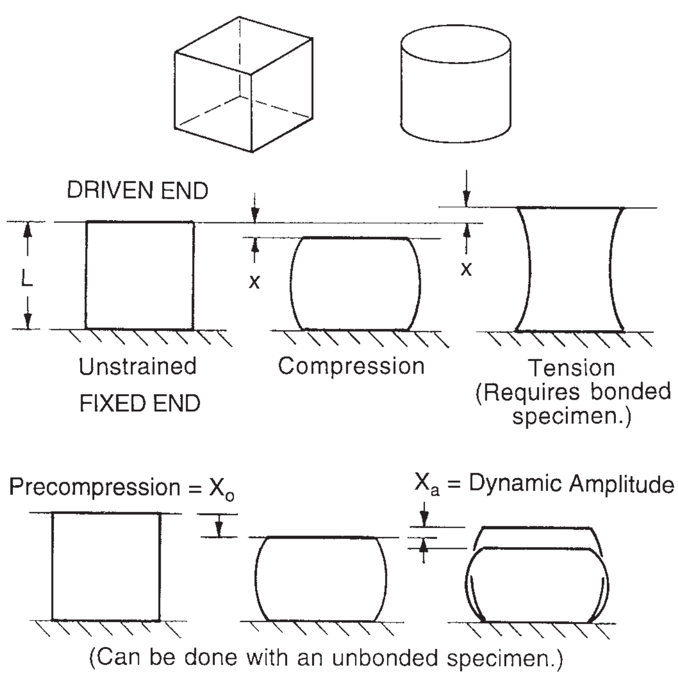


FIG. 13 Compression/Extension Specimens

9. Nonresonant Analysis Methods and Their Influences on Results

9.1 *Analyses of Nonresonant Vibratory Data*:

9.1.1 The following generic analysis methods are in wide use:

9.1.1.1 Fourier Transform,

9.1.1.2 Sine Correlation,

9.1.1.3 Perfect ellipse whose area equals the true area, height and width from actual peak-to-peak force and displacement, and

9.1.1.4 Perfect ellipse whose phase shift is defined by measured zero-crossings, height and width from actual peak-to-peak force and displacement.

9.1.2 If all materials were linear, these methods would all produce the same results. At small dynamic strains, symmetrical about zero strain, the force response waveform is essentially sinusoidal and the four methods are substantially equivalent. It is at high strains, where engineering materials are nonlinear, that they diverge in results. At high strains the resulting dynamic force is, in general, not sinusoidal, and some of the assumptions fail.

9.1.3 *Ideal Linear Case*:

9.1.3.1 Fig. 15 illustrates the ideal linear case, where both motion and force are sinusoidal. The two waveforms, when plotted against each other, produce the familiar elliptical hysteresis loop, the area of which is the energy loss per cycle. The phase angle by which the sinusoidal force leads the imposed sinusoidal motion is by definition the loss angle. (In Fig. 15 the loss angle is 35° and the loss factor tanδ is 0.7.)

9.1.3.2 From the vector relationship the mathematics are seen to be:

$$K^* = F^*/X^* = F^*_{pp}/X^*_{pp} \quad (3)$$

$$K' = K^* \cos\delta \quad (4)$$

$$K'' = K^* \sin\delta \quad (5)$$

$$\text{Energy loss per cycle} = \pi F^* X^* \sin\delta \quad (6)$$

The energy equation gives the area of the ellipse in units of the product of force and deflection, for example, newton metres or pound inches.

9.1.4 *Nonlinear Response Case*:

9.1.4.1 Fig. 16 and Fig. 17 illustrate more realistic cases. The first is for a high dynamic strain about zero mean strain. The second is for the same dynamic strain but about a high mean strain, making the nonlinearity even more pronounced. In both cases the imposed motions were sinusoidal; the resulting forces are not. The dynamic stiffnesses, if calculated using F^*_{pp} and X^*_{pp}, become highly influenced by the waveshape of the dynamic force (that is, by the "pointiness" of the peaks). Any analysis method depending on peak-to-peak measurements is sensitive to this influence.

9.1.5 *The FFT Method*:

9.1.5.1 The Fourier Transform method allows analysis of nonsinusoidal dynamic forces in a manner that minimizes the influence of force waveshape. A popular algorithm for the transform is the Fast Fourier Transform, sometimes abbreviated "FFT." In this method both the dynamic motion and force signals are digitized and then subjected to Fourier analysis. Through the transform the fundamental and harmonic components of each waveform are calculated. The fundamental is the component having the same frequency as the imposed motion.

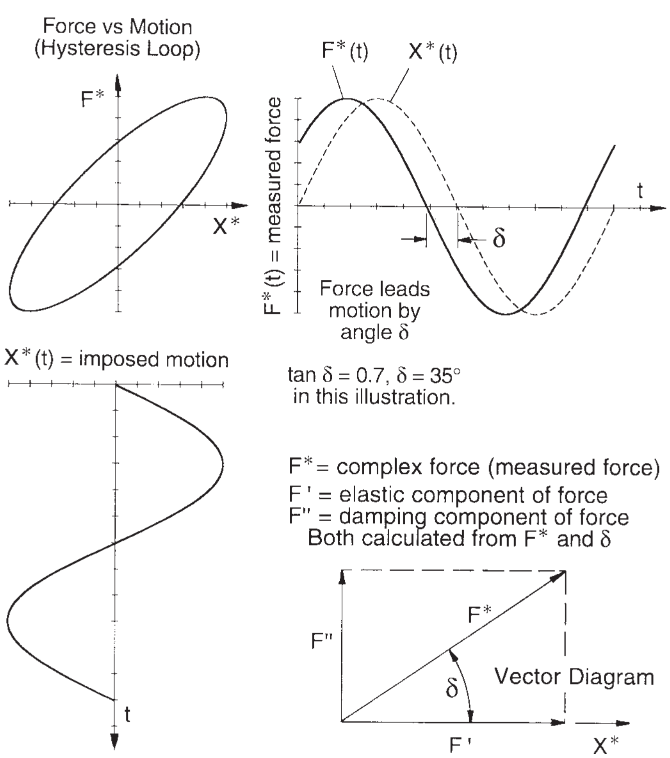


FIG. 15 Ideal Linear Case—Motion and Force Both Sinusoidal

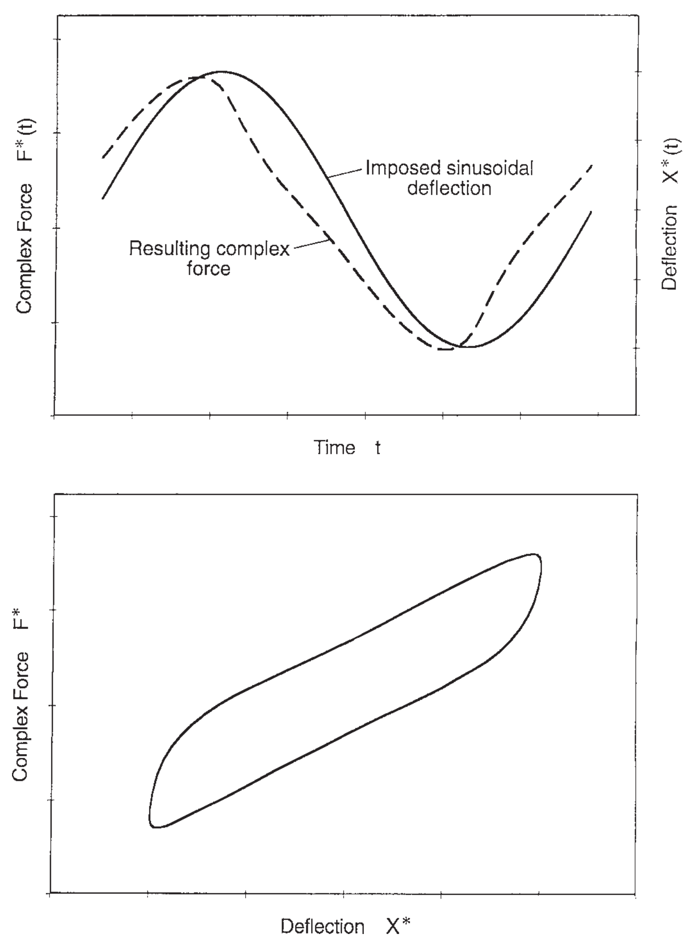


FIG. 16 Waveforms and Hysteresis Loop—Symmetrical Case, High Dynamic Strain About Mean Strain of Zero

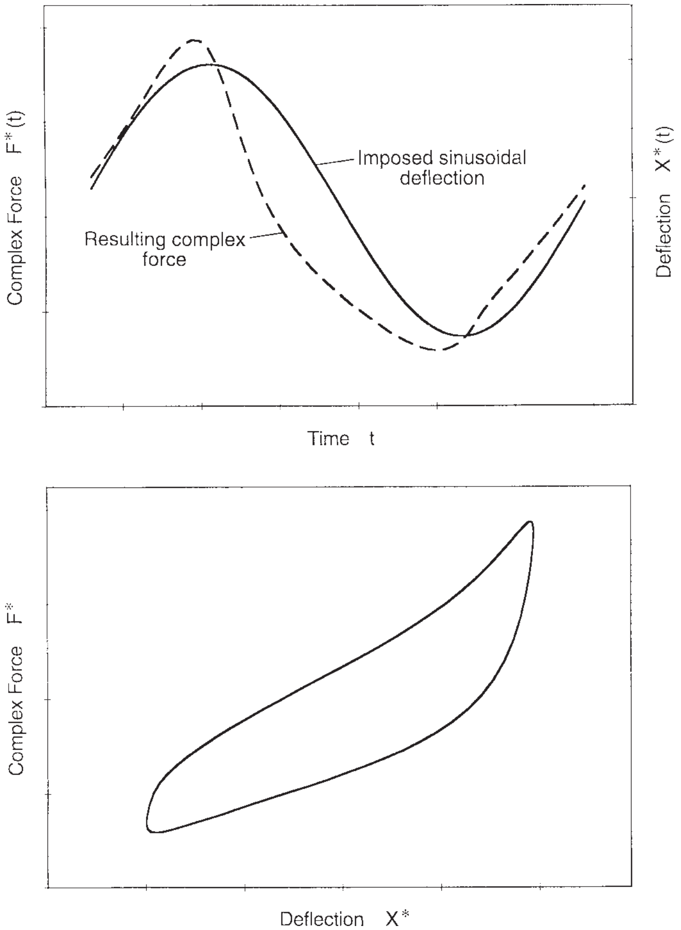


FIG. 17 Waveforms and Hysteresis Loop—Unsymmetrical Case, High Dynamic Strain About High Mean Strain

Its higher harmonics are what give the dynamic force its nonsinusoidal wave shape. The imposed motion, being sinusoidal, produces the fundamental only; its higher harmonics should be zero, or very small. When used in the analysis of elastomers, only the fundamentals are used. Since both fundamentals are sine waves, the hysteresis loop plotted from them is a perfect ellipse and the formulas in 9.1.3.2 can be used.

9.1.5.2 The areas of the loops formed by the fundamentals and by the original raw data waveforms are equal. This is because, on average, the raw data loop is as much smaller than the ellipse in some places as it is larger in others. Mathematically, the energies are the same; all the energy can be considered to be in the fundamentals. Because this is true, the loss angle is defined as the phase angle of the force fundamental component relative to the motion fundamental component.

9.1.6 *Peak-to-peak—Loss Angle Derived from Area*:

9.1.6.1 Energy per cycle can be measured by integration of the true area within the original hysteresis loop. Integration could be accomplished manually by planimeter, but is most often done by digitizing the waveforms and performing the integration in a computer. The energy per cycle thus measured is the true value.

9.1.6.2 Given this energy per cycle from integration, and the two peak-to-peak data values (F^*_{pp} and X^*_{pp}), if the assumption is made that the two waveforms are sinusoidal, an ellipse can be constructed using the mathematics of paragraph 9.1.3.2 and the illustration in Fig. 15. The construction implies a phase angle δ. If, however, the waveforms are not sinusoidal, the ellipse will be arbitrarily tall, or short, or too wide or narrow, influenced by the nonsinusoidal shapes of the waves. Since the area is one of the "givens" in the construction, the result is error in calculated phase angle. This method, therefore, when the response waveform is not sinusoidal, produces a perceived loss angle not in agreement with the Fourier method.

9.1.6.3 As explained in 9.1.4, when the response waveform is not a sine wave, stiffnesses calculated from the quotient of F^*_{pp} (t)/X^*_{pp} (t) will also disagree with those obtained from the Fourier method.

9.1.7 *Peak-to-peak—Zero-crossings Define Phase Angle*:

9.1.7.1 This method works well unless the response waveform is nonsinusoidal. Mathematically, phase has no meaning except between sine waves. Technologically, electronic circuits exist that will output a number termed "phase angle," based on the times at which two waveforms change polarity (the zero crossings). In similar manner, this angle can also be determined from oscilloscope or oscillograph displays. This angle increases with increasing damping, but in the strict sense it is not phase because one waveform (the response) is not a sine wave. In nonsymmetrical cases, such as that of Fig. 17, the results of such a circuit would be quite different, depending on whether the polarity change selected for use was from minus to plus or plus to minus.

9.1.7.2 In a system using this method, if the force response is nonsinusoidal, the angle so measured will not have the same value as the phase between fundamentals measured by the FFT. If the energy per cycle is derived from the assumption of an ellipse and peak-to-peak force and motion measurements, the energy so calculated will not agree with the FFT value. Such systems assume both waveforms are true sine waves.

9.1.8 The Fourier Transform (FFT) is the preferred method when the test equipment produces signals amenable to such analysis. It has the advantages of having good reproducibility, exact agreement of derived and actual energy loss per cycle, and minimization of influences of subtle but critical variations in waveshape. The sine-correlation method is considered to be equivalent.

9.1.9 This guide makes the tacit assumption, not always stated, that the imposed sinusoidal parameter is motion, and force is the response. In this case, it is complex force that is apt to be nonsinusoidal. The other case is used in some types of apparatus; the imposed sinusoidal parameter is force, and deflection of the specimen is the response. The comparison between results from the two methods has not been well studied. Both methods are equally possible in a servohydraulic machine.

10. Report

10.1 Because there is such a variety of methods of testing rubber, it is essential that the report state clearly the nature of the test and apparatus employed, the test specimen and its geometry, and the test conditions. A significant part of the description of test conditions involves stating the number of cycles of motion imposed, their frequency, the mean and dynamic strain amplitudes, and any time between test segments during which heat flow out of the rubber, with consequent reduction in temperature, might occur.

10.2 When measured values are plotted as functions of strain amplitude, frequency, or temperature, it is helpful to use logarithmic axes for the dynamic moduli, strain amplitude, and frequency. Linear axes better portray mean strain and temperature.

10.3 Tandel and the angle δ can be plotted on either linear or logarithmic axes. It is often convenient to include them on the same logarithmic plot as the dynamic moduli. (When logarithmic axes are used for these damping values, there is the temptation to think in terms of percentage change. This should be resisted for small damping values. The basic problem lies in the precision and accuracy of the measurement of phase angle. Careful statistical studies will guide the user in interpreting and accepting observed data.)

11. Keywords

11.1 apparatus; damping; dynamic; elastic; elastomer; guide; methods; modulus; rubber; spring rate; stiffness; testing

ANNEX

(Mandatory Information)

A1. MECHANICAL AND INSTRUMENTATION FACTORS INFLUENCING DYNAMIC MEASUREMENT

A1.1 Scope

A1.1.1 This annex covers some of the mechanical and instrumentation factors that can influence dynamic measurements of rubber and rubber-like materials. It will be helpful to list the items to be discussed, to give an overview of the matter, and then address them individually.

	Paragraph
Nonresonant Forced Vibration	A1.2
General Comments	A1.2.1
Calibration for Measurement of Force and Deflection	A1.2.2
Calibration for Measurement of Phase	A1.2.3
Statistical Studies	A1.2.8
Signal-to-Noise Ratio	A1.2.9
Frequency and Phase Response	A1.2.10
Machine Design and Transducer Location	A1.2.11
Mass and Installation of the Machine	A1.2.12
Free or Forced Resonant Vibration	A1.3
General Comments	A1.3.1
Measurement of Absolute Vibratory Motion	A1.3.2
Measurement of Relative Vibratory Motion	A1.3.3

A1.2 Nonresonant Forced Vibration

A1.2.1 *General Comments*:

A1.2.1.1 Instrumentation and analysis equipment used for measurement of dynamic forces and deflections is, in general, more complicated than that used for other tests on elastomers. As such, it can exert a greater influence on the test results. The following guidelines discuss in general terms some of the factors that should be considered when selecting equipment for this purpose.

A1.2.1.2 The frequency response of transducers and signal conditioners, and of display and analysis equipment, should be uniform over the frequency range of tests to be conducted. In addition, it is important that there be sufficient additional higher frequency response to prevent distorting the non-sinusoidal response waveforms encountered.

A1.2.1.3 Force and motion transducers should exhibit as little zero drift and scale factor change with time and temperature as possible. Zero drift appears in results as a change in mean value, and scale factor change as stiffness or modulus error. Unless the apparatus is calibrated at the time of use, the scale factor must be stable over the time period between verifications. Zero drift must be stable over the time period of the test being performed.

A1.2.1.4 Calibration will be required for the systems used to measure both force and deflection. Calibration will be required for measurement of phase. In general, verification of calibration of all three, and recalibration if necessary, should be done not less often than annually. (Calibration and verification are similar in that they use the same types of standards and procedures. Calibration involves adjustment to make the apparatus exhibit a desired scale factor, within some established acceptable tolerance. Verification involves showing that the apparatus continues to exhibit the scale factor, again within some established acceptable tolerance. Recalibration should be resorted to only if verification shows the machine to be out of tolerance.)

A1.2.2 *Calibration for Measurement of Force and Deflection*:

A1.2.2.1 True calibration and verification involves the use of standards of length and mass or force, and may involve the use of suitable transfer standards. Standards should be traceable to national standards.

A1.2.3 *Calibration for Measurement of Phase*:

A1.2.3.1 *Introduction*:

(a) (*a*) Calibration of the system for the measurement of damping requires a standard of phase. The most practical standard phase angle is zero, because it is easy to produce and, when provided by a metal spring, is stable.

(b) (*b*) The most practical physical standard of zero phase is a low damped metal spring, deflected to stresses well below the yield point. With a well designed and applied spring made of low damped material, the dynamic force and deflection signals are assumed to be in phase. Whatever phase is measured is accepted and defined as zero and used as the reference for future phase measurement. Making the system output "zero" for measurements made on the standard spring is done by adjustments in electronic filters or in software.

(c) (*c*) It is, of course, not possible for any mechanical spring to have truly zero damping. Most elastomers have inherent damping significantly greater than that of metals such as steel or aluminum. It is therefore common practice to

assume that the damping of a metal spring adequately represents zero damping. The spring must be used below any self-resonant natural ringing frequencies. The same spring(s) can be used for the statistical charting of A1.2.8.

(d) (*d*) The choice of spring type and stiffness depends on the mechanics of the system to be calibrated. In choosing a spring for use as a standard of damping, the spring's own internal resonances must be well above the range of frequencies for which it is to be used as a standard. Figs. A1.1-A1.3 show a variety of springs.

A1.2.4 *Time Interval Between Verifications*:

A1.2.4.1 Verification for measurement of phase should be performed at the same time interval as that for force and deflection.

A1.2.5 *Springs for Machines That Translate*:

A1.2.5.1 There are several possible geometric configurations of steel springs for translational machines. They differ in linearity and end constraints. Examples are: (*1*) a simple wound coil spring, (*2*) a fully machined double-helix opposed-helix coil spring, (*3*) a ring loaded across a diameter, (*4*) a variation of (*3*) in which four straight beams are loaded in fixed-fixed bending, and (*5*) a metal tube loaded axially (see *a* through *e* in Fig. A1.1).

A1.2.5.2 Commercial die springs (*a*) are the most commonly available coil springs, available in a variety of stiffnesses at low cost. In common with all simple coil springs they suffer from one distinct problem: when compressed, one end tends to rotate with respect to the other. This can cause friction between the ends of the spring and whatever flat plates are used to load it. Since the object is to have a friction-free spring, this is an immediate and obvious problem. From a practical standpoint, it has not been a serious problem; the damping is still nearly zero.

A1.2.5.3 Fully machined double opposed helices (*b*) are available, which completely eliminate this problem. They must be selected with care, because they have high stresses and may easily be yielded. They also have a relatively low natural ringing frequency because of the center mass between the two helices.

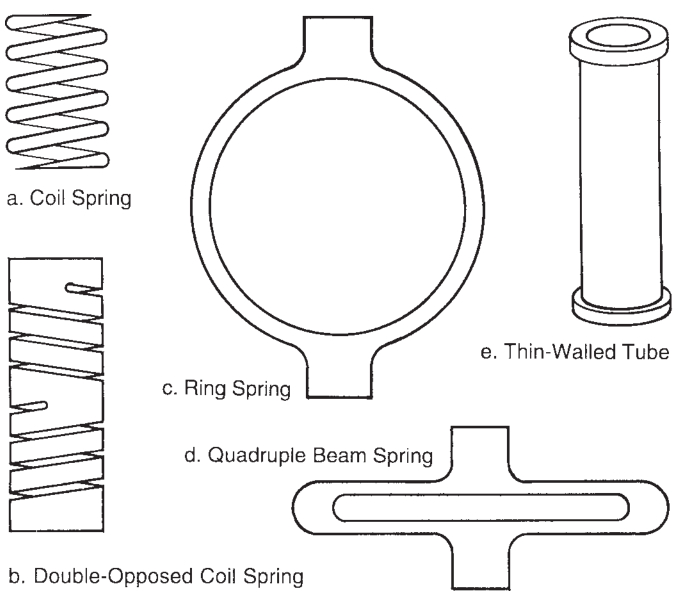


FIG. A1.1 Typical Metal Springs For Use As Standards of Damping in Translational Machines

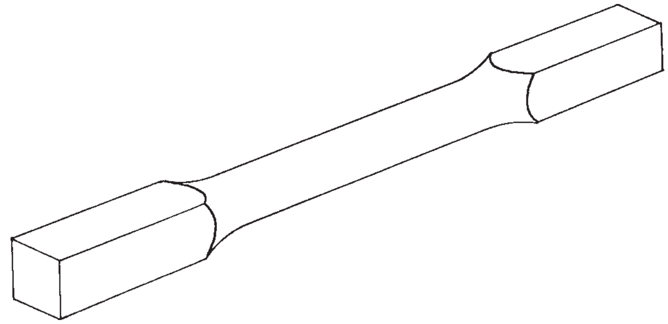


FIG. A1.2 A Torsion Spring For Use As a Standard of Damping

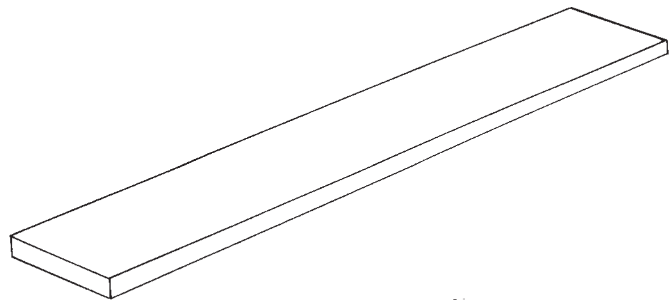


FIG. A1.3 A Flat Beam Spring For Use As a Standard of Damping in a Bending Machine

A1.2.5.4 Circular rings loaded across a diameter (*c*) are commercially available as proving rings. The same geometry, omitting the diameter measuring apparatus inside, provides an excellent spring.

A1.2.5.5 A variation of this design offers compactness, ease of manufacture, and better linearity. This is the arrangement of four fixed-fixed beams in bending (*d*).

A1.2.5.6 A thin-walled tube (*e*), loaded axially, provides a very stiff spring having the highest self-ringing frequency of all. End constraints must be handled with care to avoid yielding or buckling.

A1.2.6 *Springs for Machines That Rotate*:

A1.2.6.1 Machines that rotate require torsion springs. Clamping to the ends requires careful design to prevent slippage, which is interpreted as damping and results in errors in calibration. Alignment of the driving and ground clamps or attachments must be done carefully (see Fig. A1.2).

A1.2.7 *Springs for Machines That Bend*:

A1.2.7.1 Machines that bend the specimen might use a simple metal beam in bending, as depicted in Fig. A1.3.

A1.2.8 *Statistical Studies*:

A1.2.8.1 In test apparatus that permits it, tests repeated periodically on a spring or a stable specimen will produce stiffness and phase or damping data that can be plotted as a run chart. Once the mean has been determined, limits of variability can be established. With this as a basis, trends or abrupt departures from the line indicate possible calibration shifts, and are cause either for explanation ("probable cause") or reverification of the calibration. Such tests can be performed daily, or as experience with the statistics dictates. It should be noted that such tests on a spring or elastomeric specimen, even though they produce results within the tolerance band for the specimen, are not in any way "calibration" or "verification." They are merely an indication that there is a high probability that the calibration has not changed.

A1.2.8.2 The run chart, especially if obtained from a good spring, provides an excellent means of determining the best-case capability of the apparatus to measure both elasticity and damping. This capability is true, of course, only for the stiffness of the particular spring, and for the motion magnitude and frequency used.

A1.2.9 *Signal-to-Noise Ratio*:

A1.2.9.1 Any test apparatus should produce signal outputs of such magnitude that they exceed the inherent noise level by a sufficient margin to provide trustworthy data. The ratio of signal to noise can be enhanced through the technique of multiple-cycle averaging where this is applicable. Where analog-to-digital converters are employed, the converters must be selected to provide (*1*) sufficient samples per dynamic cycle, and (*2*) sufficient digital resolution ("counts" or "bits") to accurately define the amplitude and shape of the waveform.

A1.2.10 *Frequency and Phase Response*:

A1.2.10.1 When damping is to be measured, the difference phase between the actual physical input and the response of transducers and signal conditioners, and of display or analysis equipment, must be zero or at least matched over the operating frequency range, and ideally to ten times the operating frequency. Accurate representation of non-sinusoidal waveforms demands that the phase shift in signal conditioners, if any, be of the constant time delay type. Since in most cases the basic mechanisms of force and motion transducers are different (for example, resistive strain gage vs. inductive LVDT), it may be that the frequency and phase responses of their signal conditioners are different. The problem can be solved in two ways: (*1*) by adding a phase shift network (filter) in the signal conditioner having the least delay to make it match the other, and (*2*) by correcting for the time difference in the analysis equipment, typically in software in a computer analysis program.

A1.2.11 *Machine Design and Transducer Location*:

A1.2.11.1 *Motion Transducer*:

(a) (*a*) When possible, the motion transducer should be mounted so as to sense the deflection of the specimen alone. Practicalities of the matter may or may not permit this. Where it cannot be done, the experimenter should be aware of the influence of structure in series with the specimen. One important question is whether the deflection of the force transducer, which is really a spring, is included in the overall deflection measurement.

(b) (*b*) Two popular types of motion transducer are the linear variable differential transformer (LVDT) and the strain gaged beam spring. The latter is a spring, gaged and calibrated to measure deflection. If it is mounted directly across the specimen it inherently adds its own stiffness in parallel with the test specimen. This added stiffness, if significant, raises the perceived stiffness and reduces the apparent damping. Too soft a beam fails to follow at high frequencies; too stiff a beam may influence the results. The LVDT does not have this problem, but it is more apt to introduce problems of calibration factor because of nonlinearity.

(c) (*c*) In general, any motion transducer should be selected and mounted in ways that minimize the influences of (*1*) its own mass, (*2*) its own stiffness, if mounted to measure relative motion, and (*3*) the stiffness and mass of any electrical cables connecting them to instrumentation.

(d) (*d*) In many cases the motion transducer senses motion across an entire chain of machine elements including machine main structure, crosshead, the columns or other side supports between the main structure and crosshead, and the force transducer. Calculations or experiments, or both, should be performed on any test machine to determine its stiffness. The following formula can then be used to determine the approximate error introduced by the inclusion of the unwanted machine elements in the motion measurement. The formula is that for springs in series, and considers only elasticity; damping is not considered. Given the fact that the machine and force transducer stiffnesses are probably only approximately known, and should be orders of magnitude greater than specimen stiffness, this approach is adequate.

$$\frac{1}{K} = \frac{1}{K_s} + \frac{1}{K_m} + \frac{1}{K_f} \tag{A1.1}$$

where:

K = perceived stiffness,
K_s = specimen stiffness,
K_m = machine stiffness, and
K_f = the stiffness of the force transducer.

To use the formula it is important that the machine stiffness be constant; the load-vs-deflection curve of the machine itself must be a straight line.

(*e*) On the basis of these calculations, the user can arrive at some maximum specimen stiffness that will result in a tolerable error. Softer specimens will be measured with less error.

(*f*) This equation can be solved for specimen stiffness in terms of perceived stiffness, machine stiffness, and force transducer stiffness. Using this solution to "correct" perceived values is theoretically possible but requires careful analysis and operation well below any resonances in the system. Correction for errors exceeding a very small percentage is not recommended.

(*g*) In general, the smaller the deflection amplitude, the greater the problem in making the measurement. If the test apparatus has a primary long-stroke motion transducer, it may be advantageous to utilize a secondary auxiliary transducer having a shorter full scale range. The short range transducer can often be mounted in a better location, thereby eliminating some or all of the machine deflection as well as improving signal-to-noise ratio.

A1.2.11.2 *Force Transducer*:

(a) (*a*) Force transducers are sensitive to forces flowing through the transducer, regardless of the source. Ideally the measurement would include only forces generated by the specimen in response to imposed motion, or only forces desired to be imposed on the specimen in the case of a force-excited machine. Other forces through the transducer are

errors, and must be identified and understood. If sufficiently small, they can be ignored. Occasionally they can be corrected for.

(b) (*b*) The mass of fixturing attached to the force transducer, or of sometimes heavy metal portions of the test specimen itself, when accelerated, gives rise to forces that will be sensed by the force transducer. Accelerations can arise from: (*1*) vibration of the floor on which the test machine rests, (*2*) deliberately imposed motion of the force transducer, and (*3*) vibration of the nominally stationary part of the test machine in response to unbalanced oscillating masses within the machine itself.

(c) (*c*) If the design of the apparatus permits, the force transducer should be stationary. It should be located between the massive stationary portion of the test machine and the stationary side of the specimen. If the force transducer must be located in the moving portion of the apparatus, any fixtures should be designed to have minimum possible mass. A test run with the specimen removed will give an indication of the magnitude of the inertia forces. Such a test is best performed with a frequency sweep, to search both for inertia forces and troublesome resonances.

(d) (*d*) The imposed motion and resulting acceleration forces are all vector quantities, having phase as well as magnitude. If corrections are to be made for them, they must be carefully planned and executed. A third channel of instrumentation may be necessary, based on an accelerometer mounted on the force transducer.

(e) (*e*) The science of "mobility" and its inverse, "mechanical impedance," will be helpful in analyzing the influences of masses and motions, and the flow of forces. Mobility is the more intuitive in the study of force flow.

A1.2.12 *Mass and Installation of the Machine*:

A1.2.12.1 It may be helpful to isolate the structure of the test machine from the floor with soft mounts, thereby making the entire machine seismic. It will be isolated even though the floor moves, providing the frequency of floor vibration is significantly higher than the isolation suspension frequency. If the floor does not have troublesome vibrations of its own, it may occasionally be helpful to take advantage of the additional mass of the floor. For this reason, it is often advisable to have an isolation mount system that can be shorted out. An example is a test swept over a frequency range that includes the resonant frequency of the seismic suspension.

A1.2.12.2 The very nature of dynamic testing demands that some portion of the test apparatus move in a periodic fashion. The moving parts of the machine inescapably have mass. These may be the piston and rod in a hydraulic machine, or the crank and connecting rod in a mechanical one. Inertia forces created in response to vibration of these parts tend to cause the entire machine to vibrate. Force signals arising from this vibration may mask those from the specimen. Because inertia forces are proportional to the oscillatory acceleration, they increase in direct proportion to the mass but as the square of operating frequency. Careful consideration must be given to the overall problem. The heavier the moving parts, the larger the motions; and the higher the frequency, the greater the problem becomes. One approach to minimizing the problem, which can never be solved entirely, is to make the mass of the non-moving parts of the machine as large as feasible. The objectives are (*1*) to minimize accelerations of the" ground" side of a stationary force transducer, and (*2*) to reduce the natural frequency of the machine on its seismic suspension.

A1.3 Free or Forced Resonant Vibration

A1.3.1 *General Comments:*

A1.3.1.1 Resonant vibration involves the vibratory motion of a mass suspended or mounted upon a spring. For the purpose of this part of the guide, the "spring" is always an elastomer, and will have some damping. If the vibration is "forced," the input to the spring-mass system may be either vibration of the base or an oscillating force imposed directly on the mass. If "free" vibration, usually only motion need be measured. The dynamic motion of interest may be either "absolute" (having the earth as a reference) or "relative" (motion of one body with reference to another, both of which may be vibrating).

A1.3.2 *Measurement of Absolute Vibratory Motion*:

A1.3.2.1 Measurement of "absolute" dynamic motion requires a transducer containing an internal mass, separated from the outer case by a spring. How the sensing means within the transducer work, and whether the transducer is employed below or above its own natural frequency, determines whether the transducer is sensitive to acceleration or to velocity. Either, if used alone, measures "absolute" acceleration of velocity. Optical transducers for direct measurement of absolute dynamic displacement are possible but are expensive and rare. Electrical analog or computer integration is possible; double integration will convert instantaneous acceleration into instantaneous displacement; single integration will convert acceleration into velocity, or velocity to displacement. Integration should be used with discretion. The underlying assumption is that the waveforms are all sinusoidal, which is seldom the case.

A1.3.3 *Measurement of Relative Vibratory Motion*:

A1.3.3.1 In a shake table case, it is sometimes desirable to measure the deflection of the specimen itself. This requires measuring "relative" displacement, the instantaneous position of the mass relative to the instantaneous position of the shake table.

A1.3.3.2 The available transducers are the same as discussed before. Each has its strengths and problems.

(a) (*a*) The strain gaged beam has inherent stiffness and a natural ringing frequency. This added stiffness, if significant, raises the resonant frequency and reduces the apparent damping. Too soft a beam fails to follow at high frequencies; too stiff a beam may influence the results. The beam may have some tendency to act as an accelerometer, with introduction of spurious response.

(b) (*b*) An LVDT is more apt to add mass to the system. In general, the coil of the LVDT should be stationary, or at least be attached to the object moving the least. If resonance is to be encountered, this means the coil should be attached to the shake table or to the ground. Most LVDT coils will withstand 20g accelerations.

(c) (*c*) Subtraction of two double-integrated accelerations to obtain instantaneous relative displacement is sometimes successful, sometimes not, depending on the degree to which the input waveforms meet the criteria of being sinusoids.

(d) (*d*) In general, all the mentioned motion transducers should be selected and mounted in ways that minimize the influences of (*1*) their own mass, (*2*) their own stiffness, if mounted to measure relative motion, and (*3*) the stiffness and mass of any electrical cables connecting them to instrumentation.

APPENDIXES

(Nonmandatory Information)

X1. GUIDE TO FURTHER READING AND RELATED STANDARDS

Tong, K. N., *Theory of Mechanical Vibration*, John Wiley & Sons, 1960. (His "structural damping" is the type found in rubber. Transmissibility curves for viscous and structural damping compared. Log decrement examined.)

Snowdon, J. C., *Vibration and Shock in Damped Mechanical Systems*, John Wiley & Sons, 1968. (Snowdon starts with the concept of complex modulus.)

Ruzicka, J. E., and Derby, T. F., *Influence of Damping in Vibration Isolation*, The Shock and Vibration Information Center, United States Department of Defense, 1971. (SVM-7; Number 7 in a set of 9.) (Excellent treatment of the various kinds of damping.)

Danko, D. M., and Svarovsky, J. E., "An Application of Mini-Computers for the Determination of Elastomeric Damping Coefficients and Other Properties," (No. 730263), presented at the SAE International Automotive Engineering Congress, Detroit, MI, 1973, in *The Measurement of the Dynamic Properties of Elastomers and Elastomeric Mounts* (Symposium, January 8–12, 1973.)

Nielsen, L. E., *Mechanical Properties of Polymers*, Reinhold Publishing Corp., 1962.

Cooley, J. W., and Tukey, J. W., "An Algorithm For The Machine Calculation of Complex Fourier Series," *Mathematics Of Computation*, April 1965.

Harris, C. M., *Shock and Vibration Handbook, Third Edition*, McGraw-Hill Book Company, 1988. (Chapter 22 includes a qualitative description of Fourier analysis. Chapter 32 contains formulas for computing the first resonance frequency of metal springs of various geometries.)

X1.1 Terminology D 1566. This is the terminology standard for Committee D 11.

X2. DOUBLE-SHEAR SPECIMENS—DERIVATION OF EQUATIONS AND DESCRIPTIONS OF SPECIMENS

X2.1 Shear Modulus in a Double-shear Specimen

X2.1.1 Modulus in any specimen is always defined as the quotient of stress divided by strain. In the double-shear specimen, stress is defined as the total bond stress at the inner member. Strain is the quotient of shear deflection divided by the thickness of the rubber wall. It is assumed that the rubber wall remains constant, and that the bond area at the inner member is equal to the bond area at the outer members.

$$\text{Modulus} = \frac{\text{stress}}{\text{strain}} = G \qquad \text{(X2.1)}$$

where:

$$\text{stress} = \frac{\text{force}}{\text{area}} = \frac{F}{A} \qquad \text{(X2.2)}$$

and

$$\text{strain} = \frac{\text{deflection}}{\text{thickness}} = \frac{x}{L} \qquad \text{(X2.3)}$$

Substituting and rearranging:

$$G = \frac{FL}{Ax} = \frac{F}{x} \times \frac{L}{A} \qquad \text{(X2.4)}$$

Recognizing *F/x* as the stiffness, *K*:

$$K = \frac{F}{x} \qquad \text{(X2.5)}$$

and substituting *K* in the equation, we have:

$$G = K \times \frac{L}{A} \qquad \text{(X2.6)}$$

This is perfectly general and applies to any shear specimen.

X2.2 Fig. X2.1 shows three double-shear specimens. One is the tall rectangular design, one is square, and one circular.

X2.2.1 For the tall rectangular double shear specimen, the total area *A* equals 2(*ab*), leading to:

$$G = K \times \frac{L}{2ab} \qquad \text{(X2.7)}$$

X2.2.2 For a square cross section *A* equals $2(a^2)$ and:

$$G = K \times \frac{L}{2a^2} \qquad \text{(X2.8)}$$

X2.2.3 Finally, for a circular cross section, $A = 2(\pi d^2/4)$ and:

$$G = K \times \frac{L}{2\left(\frac{\pi d^2}{4}\right)} = K \times \frac{2L}{\pi d^2} \qquad \text{(X2.9)}$$

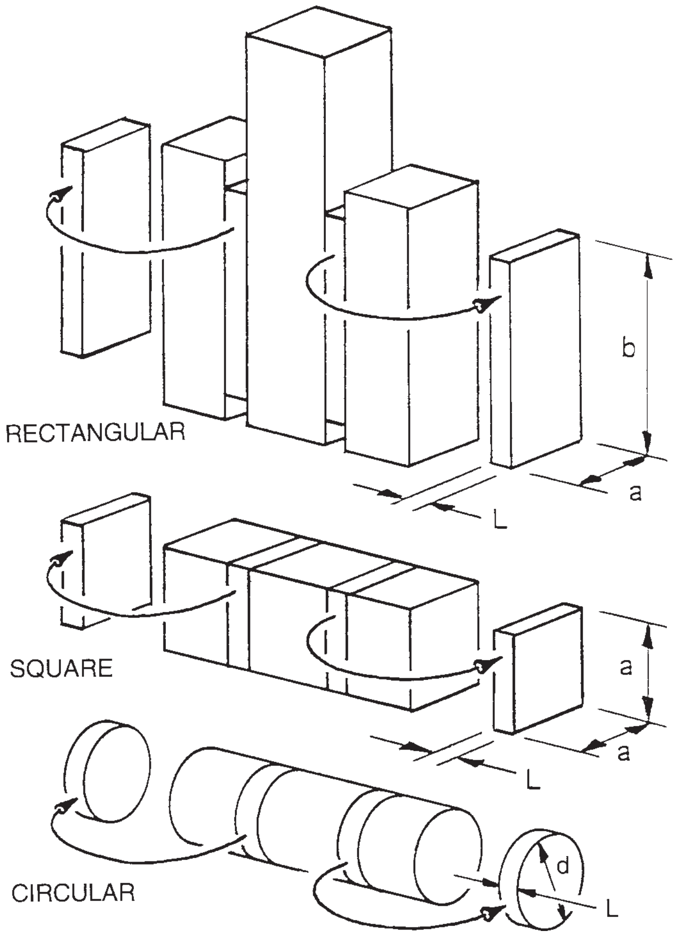
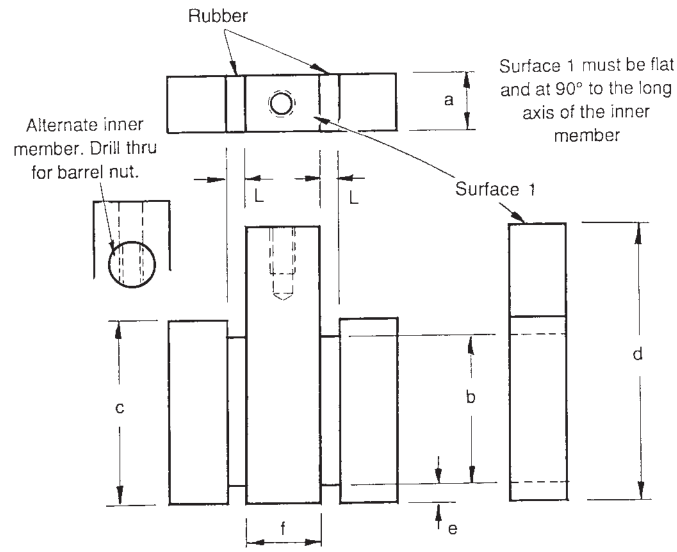


FIG. X2.1 Dimensions of Individual Rubber Elements in a Double-Shear Specimen. Rubber Elements Shown Isolated for Clarity

where:

G	=	shear modulus,
F	=	force,
x	=	deflection,
A	=	total bond area of inner member,
L	=	thickness of elastomer,
a	=	width of bond area if rectangular or square,
b	=	height of bond area if rectangular, and
d	=	diameter of bond area if circular.

X2.3 Fig. X2.2 gives specific dimensions for SI and English versions of the recommended tall rectangular double shear specimen. The two versions use commonly available sizes of bar stock for the metal parts. They are sufficiently close in size that results should be very comparable. Both have a height-to-thickness ratio of 8.0, sufficient to place the vast majority of the elastomer in shear rather than bending.



DIMENSIONS							
	CRITICAL			CONVENIENT			
	L	a	b	c	d	e	f
mm	5.00	16.00	40.00	50.0	75.0	5.0	20.0
inches	0.200	0.625	1.600	2.00	3.00	0.20	0.75

FIG. X2.2 Recommended Double-Shear Specimen, with Dimensions

X2.4 For reference purposes, DIN 53 513 specifies L = 4.0 mm, and a/L and d/L ratios of 4.0. This leads to 16.0 mm square or circular specimens, having total bond areas A of 512 and 402 square millimetres, respectively. ISO 2856 specifies ratios of $a/L = 4$ and $d/L = 2$. BS 903: Part A24 (1976) specifies that the ratio of a/L or d/L be at least 4.

X2.5 Equations X2.1 through X2.5 all assume that L is constant. It should be noted that there is an alternate way to support and deflect a double-shear specimen. The difference lies in whether or not the outer members are held a fixed distance apart (keeping the rubber wall constant), or whether the outer members are allowed to respond freely to the natural tendency to decrease the rubber wall. At high strains the difference is significant. The formulas given in this guide are for small strains, where the difference is not important. One way to achieve unconstrained thickness is illustrated in Fig. X2.3. It should also be noted that, in the quadruple-shear specimen shown, the mass of each outer member will execute free resonant vibration on the rubber sandwiches to which it is bonded. This has serious implications in setting an upper bound to useful operating frequencies with such a specimen. Also, in the quadruple-shear specimen the stiffness of the outer members must be high in order to keep them from bending, which would allow the elastomeric elements to become wedge shaped.

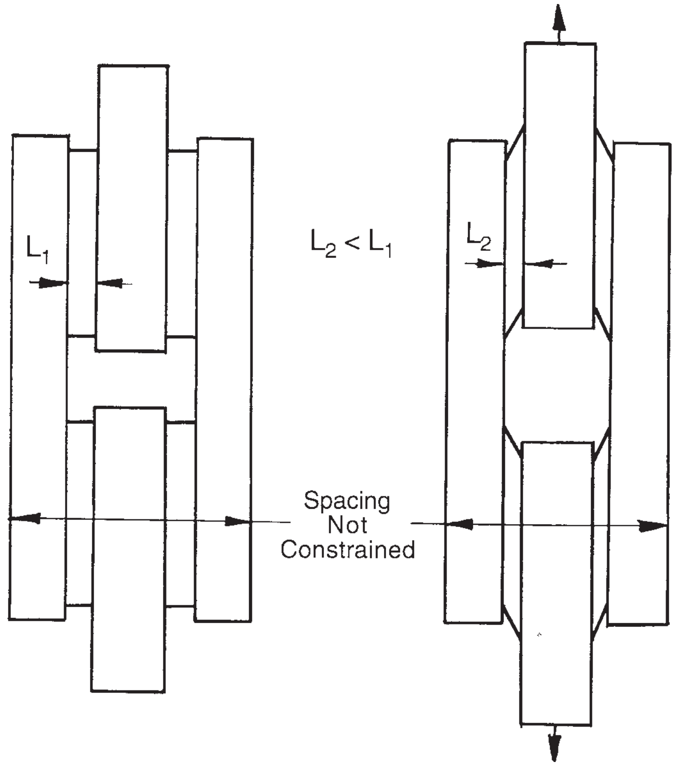


FIG. X2.3 Double (Quadruple) Shear Specimen with Unconstrained Thickness

X3. TORSION SPECIMENS: EQUATIONS AND DESCRIPTIONS OF SPECIMENS

X3.1 Rectangular Cross Section

X3.1.1 For a rectangular bar of length *L* and cross section dimensions *a* and *b*, twisted about the long axis through a dynamic angle θ and resisting with a dynamic torque *S**:

$$G^* = \frac{S^*}{\theta} \times \frac{16L}{ab^3\left[\frac{16}{3} - 3.36\frac{b}{a}\left(1 - \frac{b^4}{12a^4}\right)\right]} \quad (X3.1)$$

$$\epsilon_{s_{max}} = \frac{(3a + 1.8b)b}{16La}\left[\frac{16}{3} - 3.36\frac{b}{a}\left(1 - \frac{b^4}{12a^4}\right)\right] \times \theta \quad (X3.2)$$

X3.2 Circular Cross Section

X3.2.1 For a bar of length *L* and diameter *d*, twisted about the long axis through a dynamic angle θ and resisting with a dynamic torque *S**:

$$G^* = \frac{S^*}{\theta} \times \frac{32L}{\pi d^4} \quad (X3.3)$$

$$\epsilon_{s_{max}} = \frac{d}{2L} \times \theta \quad (X3.4)$$

These specimens are illustrated in Fig. 12.

X4. COMPRESSION/TENSION SPECIMENS: DERIVATION OF EQUATIONS AND DESCRIPTIONS OF SPECIMENS

X4.1 Compression/Tension Modulus

X4.1.1 Modulus is defined as the quotient of stress divided by strain. In a compression/tension specimen, stress is defined as the applied force divided by the contact area. Strain is defined as the change in thickness divided by the initial thickness. Force is applied normal to the contact area. Deflection is measured normal to the contact area. "Compression," "tension," and "extension" are used interchangeably to signify modulus obtained by this method.

$$\text{Modulus} = \frac{\text{stress}}{\text{strain}} = E \quad \text{(X4.1)}$$

$$\text{stress} = \frac{\text{force}}{\text{area}} = \frac{F}{A} \quad \text{(X4.2)}$$

$$\text{strain} = \frac{\text{deflection}}{\text{thickness}} = \frac{x}{L} \quad \text{(X4.3)}$$

Substituting and rearranging:

$$E = \frac{FL}{Ax} = \frac{F}{x} \times \frac{L}{A} \quad \text{(X4.4)}$$

Recognizing *F/x* as the stiffness, *K*, and substituting *K* for *F/x* in the equation, we have:

$$E = K \times \frac{L}{A} \quad \text{(X4.5)}$$

As in the derivation for the double-shear specimen in Appendix X2, this is perfectly general and applies to a tension/compression specimen of any shape.

X4.2 Fig. X4.1 shows three basically button-shaped specimens: circular, square, and rectangular.

X4.2.1 For the circular button of diameter *d*:

$$E = K \times \frac{L}{\left(\frac{\pi d^2}{4}\right)} = K \times \frac{4L}{\pi d^2} \quad \text{(X4.6)}$$

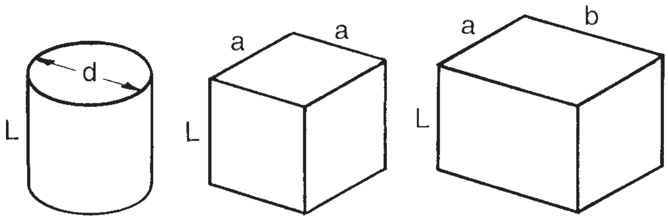


FIG. X4.1 Circular, Square, and Rectangular Compression/Extension Specimens

For a square cross section having width *a*:

$$E = K \times \frac{L}{a^2} \quad \text{(X4.7)}$$

For a rectangular cross section having section dimensions *a* and *b* :

$$E = K \times \frac{L}{ab} \quad \text{(X4.8)}$$

where:
- E = compression or tension modulus,
- F = force,
- x = deflection,
- A = contact area at one face (assumed equal),
- L = initial thickness of the elastomer,
- a = width of one side if square, or of first side if rectangular,
- b = width of second side if rectangular, and
- d = diameter if circular.

X4.2.2 Note that all three equations are of the same form as those for the double-shear specimens, but are twice the magnitude. This is because the area for the double-shear is twice a^2 or twice *ab* or twice $\pi d^2/4$ because the design has two bond areas. The compression/tension design has only one contact area.

X4.3 For reference purposes, dimensions for circular specimens found in some commonly used standards are given in Table X4.1.

TABLE X4.1 Dimensions for Circular Specimens

NOTE 1—ISO 2856 specifies only that for the square specimen, $a = 2L$ and $d = L$.

Standard	L	d	$4L/\pi d^2$
Test Methods D 945 (Yerzley)	12.5 mm 0.50 in.	19.5 mm 0.75 in.	0.041855 mm^{-1} 1.3177 in.$^{-1}$
DIN 53 513	10.0 mm	10.0 mm	0.12732 mm^{-1}

X5. FREE RESONANT VIBRATION—EQUATIONS FOR LOG DECREMENT AND STIFFNESS

X5.1 Equations for Log Decrement and Stiffness

X5.1.1 The following material relates to the introduction provided in 8.3, and provides guidance and equations for performing the test.

X5.1.2 Log decrement Δ is classically defined as the natural logarithm of the amplitudes of two successive peaks of the same sign in the decaying wave. Fig. X5.1 illustrates how to acquire the needed data from a typical wave. Any two successive peaks may be used. They could be positive or negative. Eq X5.1 calculates Δ from the data.

$$\Delta = \ln \frac{a_1}{a_2} \qquad \text{(X5.1)}$$

X5.1.3 Because it is difficult to establish the zero-amplitude reference line, Δ is more easily measured employing an alternate formula using peak-to-peak amplitudes. It is also possible to allow more than one cycle to occur between the two measurements. Eq X5.2 calculates Δ from the peak-to-peak data in Fig. X5.2.

$$\Delta = \frac{1}{k} \ln \frac{a_n}{a_{n+k}} \qquad \text{(X5.2)}$$

The symbols are as depicted in Fig. X5.2.

X5.1.4 Log decrement can be converted to loss factor or to loss angle by these approximate relationships:

$$\tan\delta = \frac{\Delta}{\pi} \qquad \text{(X5.3)}$$

$$\delta = \arctan\left(\frac{\Delta}{\pi}\right) \qquad \text{(X5.4)}$$

For the waves shown, Δ = 0.31, tanδ = 0.10 and the phase angle δ = 5.7 degrees.

X5.1.5 Eq X5.5 and Eq X5.6 give the derivation of the expression for the elastic stiffness K' in terms of period P and supported mass M (see Fig. X5.3).

X5.1.5.1 Starting with the equation for the undamped natural frequency:

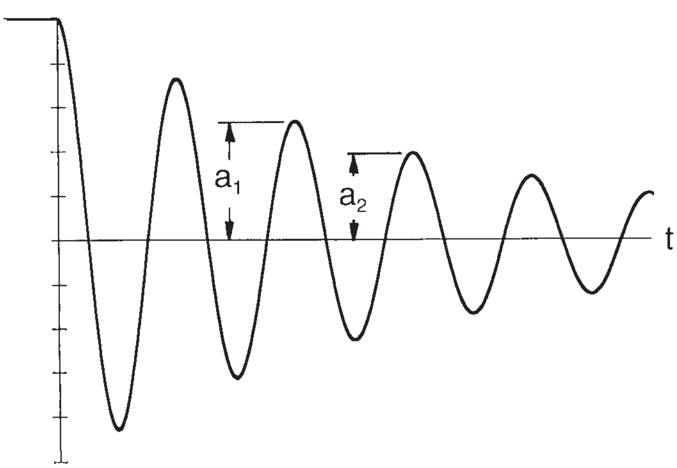


FIG. X5.1 Single Amplitude Data from Successive Peaks

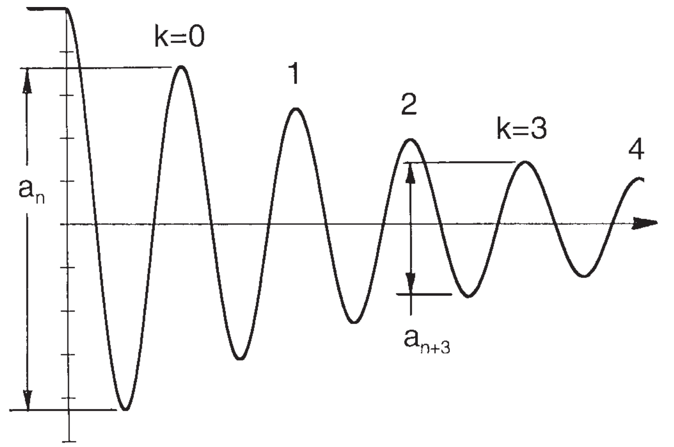


FIG. X5.2 Peak-to-peak Data from Peaks Separated by More Than One Cycle

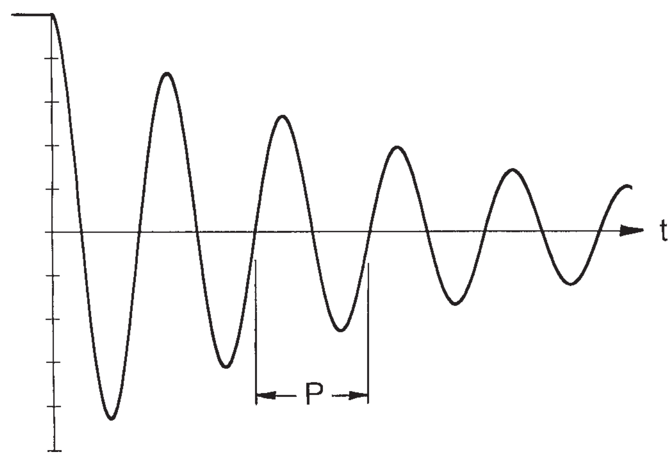


FIG. X5.3 Obtaining the Period from the Decay Wave

$$f_n = \frac{1}{P} = \frac{1}{2\pi}\sqrt{\frac{K'}{M}} \qquad \text{(X5.5)}$$

Rearranging and solving for K':

$$K' = \frac{4\pi^2 M}{P^2} = 4\pi^2 f_n^2 M \qquad \text{(X5.6)}$$

X5.1.6 Eq X5.7 and Eq X5.8 give a similar derivation for the torsion case, giving the elastic torsion stiffness R' in terms of period and mass moment of inertia I .

$$f_n = \frac{1}{P} = \frac{1}{2\pi}\sqrt{\frac{R'}{I}} \qquad \text{(X5.7)}$$

Rearranging and solving for R':

$$R' = \frac{4\pi^2 I}{P^2} = 4\pi^2 f_n^2 I \qquad \text{(X5.8)}$$

X5.1.7 The applicable units for Eqs X5.5 through X5.8 are given in Table X5.1

TABLE X5.1 Units for Equations X5.5 through X5.8

Symbol	Physical Quantity	SI Units	English Units
f	frequency	Hz	Hz
P	period	seconds	seconds
K′	translational elastic stiffness	Newton/metre	lbf/in.
M	mass	kg	
W	weight		lb
g	accel of gravity	m/sec^2	in./sec^2
R	torsional elastic stiffness	N · m/radian	lbf · in./radian
I	mass moment of inertia	kg·m^2	lb · in.·sec^2

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X6. OBTAINING LOSS FACTOR AND ELASTIC STIFFNESS FROM TRANSMISSIBILITY CURVES

X6.1 Introduction

X6.1.1 Paragraph 8.4 described forced resonant vibration and its limitations in measuring loss factor and elastic stiffness (and hence elastic modulus). This appendix gives detailed instructions for obtaining these values from transmissibility curves. Both hysteretic and viscous models are covered, the latter for academic completeness and for the occasional case where a viscous damper may be used with an elastomeric spring.

X6.2 Data

X6.2.1 Forced resonant tests produce a transmissibility curve as their "output." To use the curve to obtain damping values and elastic stiffness, the following must be known:

X6.2.1.1 Does the curve show relative or absolute transmissibility?

X6.2.1.2 Was the test run at constant amplitude? It might have been at constant velocity or acceleration.

X6.2.1.3 Are the input and output given as single-amplitude or double-amplitude? Vibration amplitudes are often given as peak-to-peak (double amplitude), whereas velocity and acceleration is usually given as single-peak (single amplitude).

X6.2.1.4 Is damping assumed to follow the hysteretic or the viscous model?

X6.2.2 The answers to X6.2.1.1 through X6.2.1.3 must be known if the deflection across the specimen, and hence the strain in the elastomer, is to be ascertained. The instructions following assume that for the question in X6.2.1.2, the test was performed at constant table amplitude.

X6.3 Determining the Model

X6.3.1 It is for the experimenter to determine which model best describes the material being measured. For most elastomeric compounds, the hysteretic is the better of the two. The determination can best be accomplished by extending the frequency of the experiment to well above resonance (to 100 times the resonant frequency if possible). The slope of the absolute transmissibility curve provides the needed data: if the slope is 12 dB/octave the damping follows the hysteretic model; if 6 dB/octave, the damping follows the viscous model. In practice, the slope will not be exactly 12 or 6, but will be close to one or the other. Fig. 6 shows these slopes.

X6.4 Measuring the Amount of Damping

X6.4.1 In general, the greater the damping, the less the magnitude of maximum transmissibility, T_{max}. For small damping, the relationships are approximately these:

$$\eta = 1/T_{max} \quad (X6.1)$$

$$\zeta = 1/(2T_{max}) \quad (X6.2)$$

Neither relationship is true for large damping. Fig. X6.1 and Fig. X6.2 show η and ζ plotted against the reciprocal of T_{max}. It is apparent that the relationships are not linear.

X6.4.2 To obtain the damping value (using the curve for the appropriate model), obtain T_{max} from the transmissibility curve, calculate its reciprocal, and determine the value of η or ζ from the curve.

X6.4.3 The curves also point out that for the hysteretic model T_{max} has the same magnitude for both absolute and

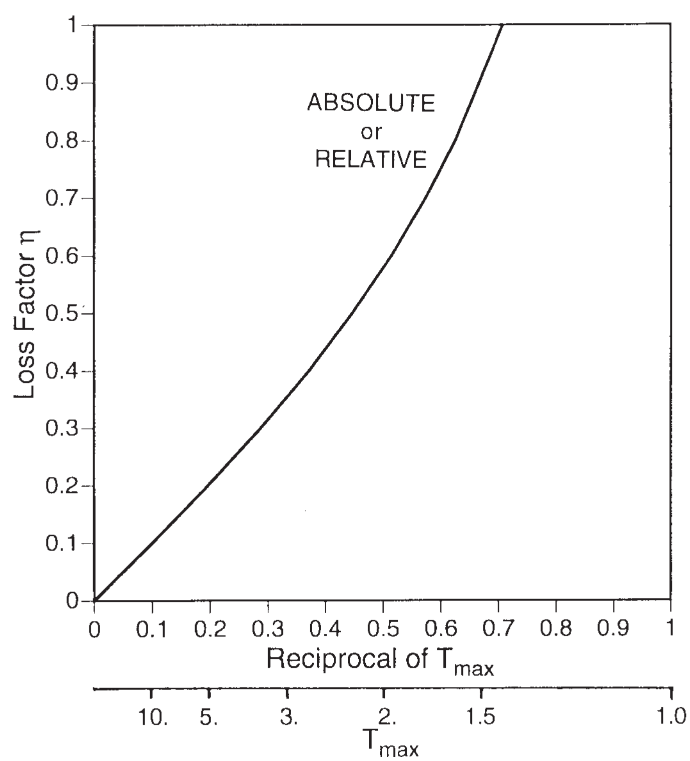

FIG. X6.1 Loss Factor η Versus Reciprocal of T_{max}(Hysteretic Damping Model)

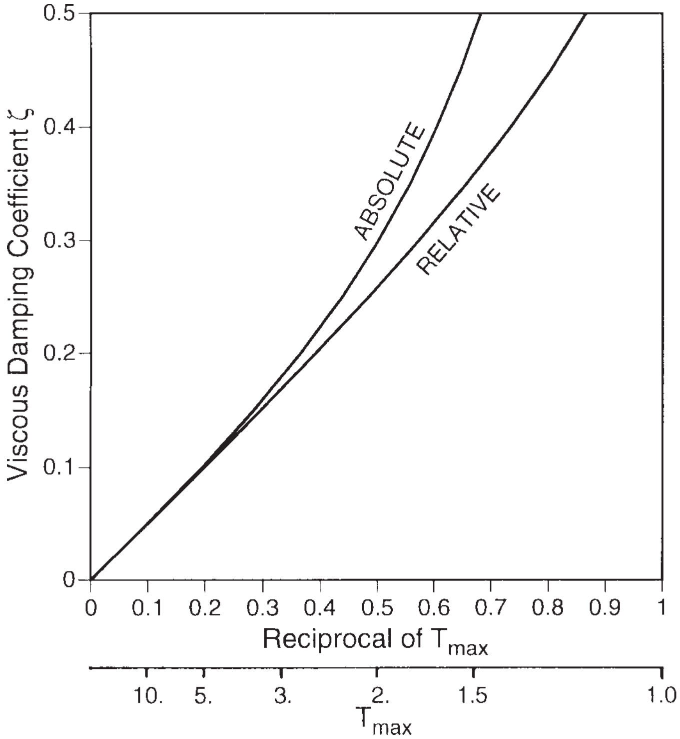

FIG. X6.2 Viscous Damping Coefficient ζ Versus Reciprocal of T_{max} (Viscous Damping Model)

relative transmissibility. For viscous damping the values differ for the two transmissibilities.

X6.5 Determining the Undamped Natural Frequency

X6.5.1 The transmissibility experiment provides the magnitude of T_{max}, the frequency at which T_{max} occurs, and a decision as to the damping model. From X6.4 we have deduced magnitudes of η or ζ, using curve X6.1 or X6.2. To be able to calculate the elastic stiffness of the specimen it will be necessary first to determine the undamped natural frequency f_n.

X6.5.2 From Curves Fig. X6.3 and Fig. X6.4, select the curve for the appropriate model. Determine the magnitude of β_{max} for the value of η or ζ found in Fig. X6.4. Divide the frequency at which T_{max} occurred, by β_{max}, to obtain the undamped natural frequency f_n.

X6.6 Calculating the Elastic Stiffness

X6.6.1 The elastic stiffness is calculated using equation X6.4, which is derived from the classic equation for undamped natural frequency:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K'}{M}} \quad \text{(X6.3)}$$

Rearranging and solving for K':

$$K' = 4\pi^2 f_n^2 M \quad \text{(X6.4)}$$

X6.7 β_{max} Frequency—Models Compared

X6.7.1 Peak absolute transmissibility for the hysteretic model occurs at the undamped natural frequency. The other three maximum transmissibilities occur at frequencies other than β = 1.

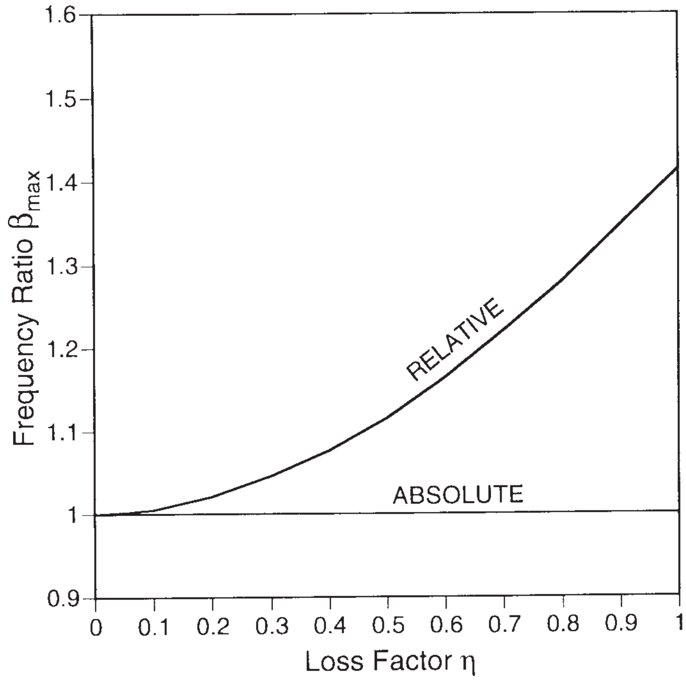

FIG. X6.3 Frequency Ratio β_{max} at Peak Transmissibility, Absolute and Relative (Hysteretic Damping Model)

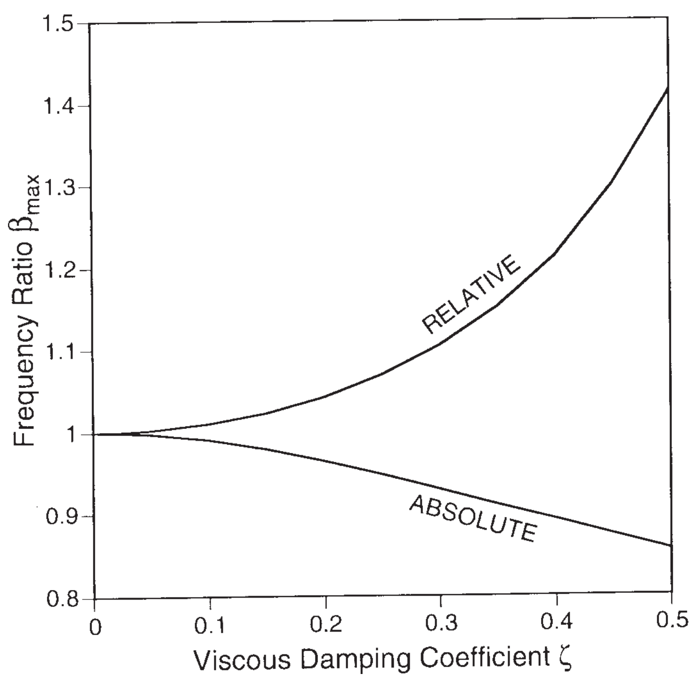

FIG. X6.4 Frequency Ratio β_{max} at Peak Transmissibility, Absolute and Relative (Viscous Damping Model)

X6.8 Phase—Models Compared

X6.8.1 Fig. 5 and Fig. 7 point out an important fact about the phase angles at which peak transmissibility occurs. For neither model does peak absolute or relative transmissibility occur at a phase angle of 90 degrees. The "90 degree phase shift" so widely remembered is for relative phase only; it defines the undamped natural frequency for relative transmissibility, not absolute.

X6.9 Dynamic Deflection Across the Specimen

X6.9.1 Measurement of the deflection across the specimen can be done either directly or indirectly. The direct method is to utilize a motion transducer mounted on either the table or the mounted mass and having its probe attached to the other. Two motion transducers can be employed, both mounted on a seismic base, one sensing the table and the other the mounted mass. The indirect method utilizes accelerometers, one on the table and the other on the mounted mass, and employing double integration to convert the acceleration signals to motion. Double integration should be used with caution, taking care to avoid distortion of the signals in either magnitude or phase, and should be avoided if the original signals are not sine waves.