ASTM

Designation: D 6029 – 96

Standard Test Method (Analytical Procedure) for Determining Hydraulic Properties of a Confined Aquifer and a Leaky Confining Bed with Negligible Storage by the Hantush-Jacob Method[1]

This standard is issued under the fixed designation D 6029; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This test method covers an analytical procedure for determining the transmissivity and storage coefficient of a confined aquifer and the leakance value of an overlying or underlying confining bed for the case where there is negligible change of water in storage in a confining bed. This test method is used to analyze water-level or head data collected from one or more observation wells or piezometers during the pumping of water from a control well at a constant rate. With appropriate changes in sign, this test method also can be used to analyze the effects of injecting water into a control well at a constant rate.

1.2 This analytical procedure is used in conjunction with Test Method D 4050.

1.3 *Limitations*—The valid use of the Hantush-Jacob method is limited to the determination of hydraulic properties for aquifers in hydrogeologic settings with reasonable correspondence to the assumptions of the Theis nonequilibrium method (Test Method D 4106) with the exception that in this case the aquifer is overlain, or underlain, everywhere by a confining bed having a uniform hydraulic conductivity and thickness, and in which the gain or loss of water in storage is assumed to be negligible, and that bed, in turn, is bounded on the distal side by a zone in which the head remains constant. The hydraulic conductivity of the other bed confining the aquifer is so small that it is assumed to be impermeable (see Fig. 1).

1.4 The values stated in SI units are to be regarded as standard.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

D 653 Terminology Relating to Soil, Rocks, and Contained Fluids[2]

D 4050 Test Method (Field Procedure) for Withdrawal and Injection Well Tests for Determining Hydraulic Properties of Aquifer Systems[2]

D 4106 Test Method (Analytical Procedure) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Theis Nonequilibrium Method[2]

D 6028 Test Method (Analytical Procedure) for Determining Hydraulic Properties of a Confined Aquifer Taking Into Consideration Storage of Water in Leaky Confining Beds by the Modified Hantush Method[3]

3. Terminology

3.1 *Definitions:*

3.1.1 *aquifer, confined, n*—an aquifer bounded above and below by confining beds and in which the static head is above the top of the aquifer.

3.1.2 *aquifer, unconfined, n*—an aquifer is unconfined where it has a water table.

3.1.3 *coefficient of leakage, n*—see *leakance*.

3.1.4 *confining bed, n*—a hydrogeologic unit of less permeable material bounding one or more aquifers.

3.1.5 *control well, n*—well by which the head and flow in the aquifer is changed, for example, by pumping, injection, or change of head.

3.1.6 *drawdown, n*—vertical distance the static head is lowered due to the removal of water.

3.1.7 *head, n*—see *head, static*.

3.1.8 *head, static, n*—the height above a standard datum of the surface of a column of water (or other liquid) that can be supported by the static pressure at a given point.

3.1.9 *hydraulic conductivity, n*—(field aquifer test) the volume of water at the existing kinematic viscosity that will move in a unit time under a unit hydraulic gradient through a unit area measured at right angles to the direction of flow.

3.1.10 *leakance, n*—the ratio of the vertical hydraulic conductivity of a confining bed to its thickness.
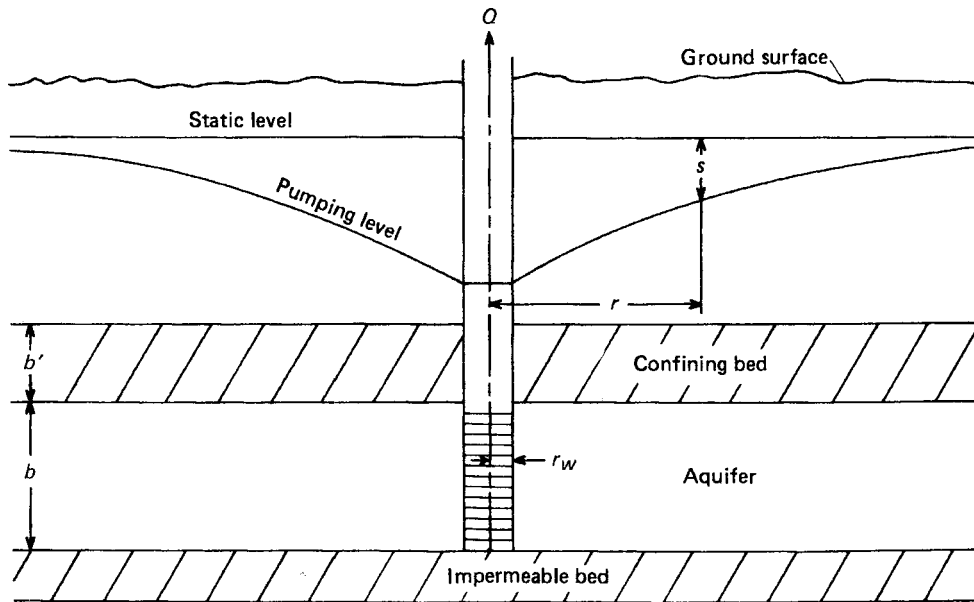
3.1.11 *observation well, n*—a well open to all or part of an aquifer.

FIG. 1 Cross Section Through a Discharging Well in a Leaky Aquifer (from Reed (1)).[4] The Confining and Impermeable Bed Locations Can Be Interchanged

3.1.12 *piezometer, n*—a device used to measure static head at a point in the subsurface.

3.1.13 *specific storage, n*—the volume of water released from or taken into storage per unit volume of the porous medium per unit change in head.

3.1.14 *storage coefficient, n*—the volume of water an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head.

3.1.14.1 *Discussion*—For a confined aquifer, the storage coefficient is equal to the product of the specific storage and aquifer thickness. For an unconfined aquifer, the storage coefficient is approximately equal to the specific yield.[4]

3.1.15 *transmissivity, n*—the volume of water at the prevailing kinematic viscosity that will move in a unit time under a unit hydraulic gradient through a unit width of the aquifer.

3.1.16 For definitions of other terms used in this test method, see Terminology D 653.

3.2 *Symbols:Symbols and Dimensions:*

3.2.1 *K*—hydraulic conductivity of the aquifer $[LT^{-1}]$.

3.2.1.1 *Discussion*—The use of the symbol *K* for the term hydraulic conductivity is the predominant usage in groundwater literature by hydrogeologists, whereas the symbol *k* is commonly used for this term in soil and rock mechanics and soil science.

3.2.2 *K'*—vertical hydraulic conductivity of the confining bed through which leakage can occur $[LT^{-1}]$.

3.2.3 *L(u,v)*—leakance function of *u,v* [*nd*]; equal to *W(u,r/B)*.

3.2.4 *Q*—discharge $[L^3T^{-1}]$.

3.2.5 $S = bS_S$—storage coefficient [*nd*].

3.2.6 S_s—specific storage of the aquifer $[L^{-1}]$.

3.2.7 S'_s—specific storage of the confining bed $[L^{-1}]$.

3.2.8 *T*—transmissivity $[L^2T^{-1}]$.

3.2.9 $u = \dfrac{r^2 S}{4Tt}$ [*nd*].

3.2.10 *W(u,r/B)*—well function for leaky aquifer systems with negligible storage changes in confining beds [*nd*].

3.2.11 *b*—thickness of aquifer [*L*]. *b'*—thickness of the confining bed through which leakage can occur [*L*].

3.2.12 *r*—radial distance from control well [*L*].

3.2.13 r_c—radius of the control well casing, or hole if uncased [*L*].

3.2.14 *s*—drawdown [*L*].

3.2.15 $v = \dfrac{r}{2B} = \dfrac{r}{2}\sqrt{\dfrac{K'}{Tb'}}$, *v*—defined by Eq 7 [*nd*].

3.2.16 $B = \sqrt{\dfrac{Tb'}{K'}}$ [*L*].

3.2.17 *t*—time since pumping or injection began [*T*].

3.2.18 $K_0(x)$—zero-order modified Bessel function of the second kind [*nd*].

3.2.19 $\beta = \dfrac{r}{4b}\sqrt{\dfrac{K'S'_S}{KS_S}}$

4. Summary of Test Method

4.1 This test method involves pumping a control well that is fully screened through the confined aquifer and measuring the water-level response in one or more observation wells or piezometers. The well is pumped at a constant rate. The water-level response in the aquifer is a function of the transmissivity and storage coefficient of the aquifer and the leakance coefficient of a confining bed. The other confining bed is assumed to be impermeable. Alternatively, the test method can be performed by injecting water at a constant rate into the control well. Analysis of buildup of water level in response to injection is similar to analysis of drawdown of the water level in response to withdrawal in a confined aquifer. The water-level response data may be analyzed in two ways. The time variation of the water-level response in any one well can be

---

[4] The boldface numbers in parentheses refer to a list of references at the end of this test method.

analyzed using one set of type curves, or the water-level responses measured at the same time but in observation wells at different distances from the control well can be analyzed using another set of type curves.

4.2 *Solution*—Hantush and Jacob (**2**) give two mathematically equivalent expressions for the solution which can be written as follows:

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{1}{z}\,exp\left(-z - \frac{r^2}{4B^2 z}\right)dz \qquad (1)$$

where z is the variable of integration and

$$s = \frac{Q}{4\pi T}\left[2K_0\left(\frac{r}{B}\right) - \int_{\frac{r^2}{4B^2 u}}^\infty \frac{1}{z}\,exp\left(-z - \frac{r^2}{4B^2 z}\right)dz\right] \qquad (2)$$

where:

$$u = \frac{r^2 S}{4Tt} \qquad (3)$$

$$B^2 = \frac{Tb'}{K'} \qquad (4)$$

4.2.1 Because a closed-form expression of the integrals that appear in Eq 1 or Eq 2 are not known, Hantush and Jacob developed equivalent expressions that involve infinite series that can be numerically evaluated. The infinite series for Eq 1 converges more rapidly for early times and the infinite series for Eq 2 converges more rapidly for late times.

4.2.2 Hantush (**3**) expressed Eq 1 and Eq 2 as follows:

$$s = \frac{Q}{4\pi T}W\left(u, \frac{r}{B}\right) \qquad (5)$$

where $W\left(u, \frac{r}{B}\right)$ was called the well function for leaky systems. Hantush tabulated values of this function for a practical range of the parameters u and $\frac{r}{B}$.

4.2.3 Cooper (**4**) opted to express the Hantush-Jacob solution in the following form:

$$s = \frac{Q}{4\pi T}L(u, v) \qquad (6)$$

where Cooper's v = Hantush's $\frac{r}{2B}$

or

$$v = \frac{r}{2B} = \frac{r}{2\sqrt{\frac{Tb'}{K'}}} \qquad (7)$$

4.2.4 Cooper prepared two families of type curves. One set of Cooper's curves allow the head changes as a function of time at a fixed distance to be analyzed for the aquifer parameters, and the other set of curves allow the head changes at different distances at some fixed time to be analyzed.

5. Significance and Use

5.1 *Assumptions*:

5.1.1 The control well discharges at a constant rate, Q.

5.1.2 The control well is of infinitesimal diameter and fully penetrates the aquifer.

5.1.3 The aquifer is homogeneous, isotropic, and areally extensive.

5.1.4 The aquifer remains saturated (that is, water level does not decline below the top of the aquifer).

5.1.5 The aquifer is overlain, or underlain, everywhere by a confining bed having a uniform hydraulic conductivity and thickness. It is assumed that there is no change of water storage in this confining bed and that the hydraulic gradient across this bed changes instantaneously with a change in head in the aquifer. This confining bed is bounded on the distal side by a uniform head source where the head does not change with time.

5.1.6 The other confining bed is impermeable.

5.1.7 Leakage into the aquifer is vertical and proportional to the drawdown, and flow in the aquifer is strictly horizontal.

5.1.8 Flow in the aquifer is two-dimensional and radial in the horizontal plane.

5.2 The geometry of the well and aquifer system is shown in Fig. 1.

5.3 *Implications of Assumptions*:

5.3.1 Paragraph 5.1.1 indicates that the discharge from the control well is at a constant rate. Section 8.1 of Test Method D 4050 discusses the variation from a strictly constant rate that is acceptable. A continuous trend in the change of the discharge rate could result in misinterpretation of the water-level change data unless taken into consideration.

5.3.2 The leaky confining bed problem considered by the Hantush-Jacob solution requires that the control well has an infinitesimal diameter and has no storage. Abdul Khader and Ramadurgaiah (**5**) developed graphs of a solution for the drawdowns in a large-diameter control well discharging at a constant rate from an aquifer confined by a leaky confining bed. Fig. 2 (Fig. 3 of Abdul Khader and Ramadurgaiah (**5**))
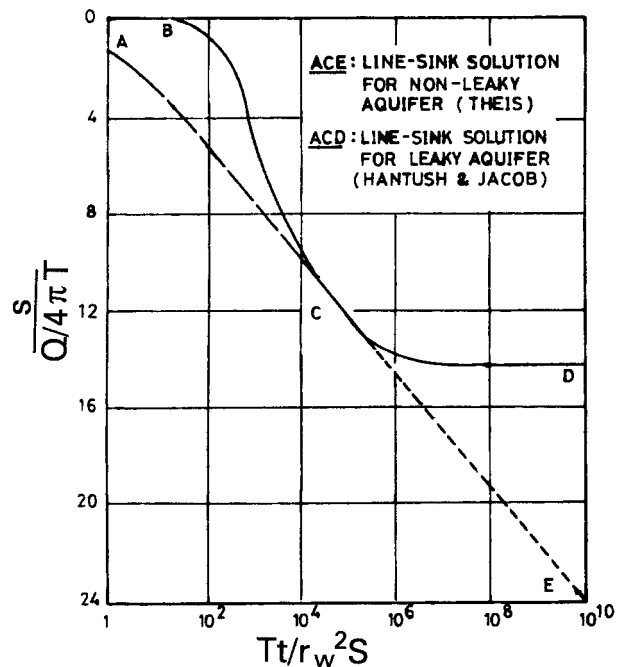
Curve *BCE*: Solution for well of large diameter in a non-leaky aquifer (Papadopulos and Cooper (**6**)).

Curve *BCD*: Solution for well of large diameter in a leaky aquifer (Abdul Khader and Ramadurgaiah).

FIG. 2 Time—Drawdown Variation in the Control Well for
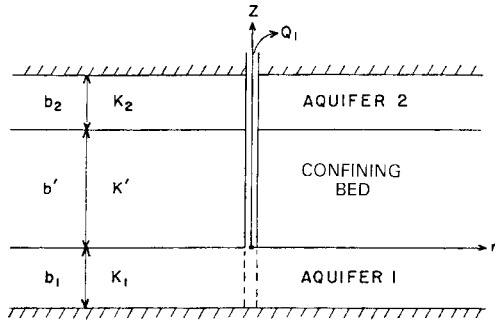$S = \delta = 10^{-3}$**(from Abdul Khader and Ramadurgaiah (5))**

FIG. 3 Schematic Diagram of Two-Aquifer System

gives a graph showing variation of dimensionless drawdown with dimensionless time in the control well assuming the aquifer storage coefficient, $S = 10^{-3}$, and the leakage parameter, $\frac{r_w}{B} = 10^{-3}$. Note that at early dimensionless times the curve for a large-diameter well in a non-leaky aquifer (BCE) and in a leaky aquifer (BCD) are coincident. At later dimensionless times, the curve for a large diameter well in a leaky aquifer coalesces with the curve for an infinitesimal diameter well (ACD) in a leaky aquifer. They coalesce about one logarithmic cycle of dimensionless time before the drawdown becomes sensibly constant. For a value of r_w/B smaller than 10^{-3}, the constant drawdown (D) would occur at a greater value of dimensionless drawdown and there would be a longer period during which well-bore storage effects are negligible (the period where ACD and BCD are coincident) before a steady drawdown is reached.

For values of $\frac{r_w}{B}$ greater than 10^{-3}, the constant drawdown (D) would occur at a smaller value of drawdown and there would be a shorter period of dimensionless time during which well-storage effects are negligible (the period where ACD and BCD are coincident) before a steady drawdown is reached. Abdul Khader and Ramadurgaiah (5) present graphs of dimensionless time versus dimensionless drawdown in a discharging control well for values of $S = 10^{-1}$, 10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5} and $r_w/B = 10^{-2}$, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, and 0. These graphs can be used in an analysis prior to the aquifer test making use of estimates of the hydraulic properties to estimate the time period during which well-bore storage effects in the control well probably will mask other effects and the drawdowns would not fit the Hantush-Jacob solution.

5.3.2.1 The time required for the effects of control-well bore storage to diminish enough that drawdowns in observation wells should fit the Hantush-Jacob solution is less clear. But the time adopted for when drawdowns in the discharging control well are no longer dominated by well-bore storage affects probably should be the minimum estimate of the time to adopt for observation well data.

5.3.3 The assumption that the aquifer is bounded, above or below, by a leaky layer on one side and a nonleaky layer on the other side is not likely to be entirely satisfied in the field. Neuman and Witherspoon (7, p. 1285) have pointed out that because the Hantush-Jacob formulation uses water-level change data only from the aquifer being pumped (or recharged) it can not be used to distinguish whether the leaking beds are

above or below (or from both sides) of the aquifer. Hantush (8) presents a refinement that allows the parameters determined by the aquifer test analysis to be interpreted as composite parameters that reflect the combined effects of overlying and underlying confined beds. Neuman and Witherspoon (7) describe a method to estimate the hydraulic properties of a confining layer by using the head changes in that layer.

5.3.4 The Hantush-Jacob theoretical development requires that the leakage into the aquifer is proportional to the drawdown, and that the drawdown does not vary in the vertical in the aquifer. These requirements are sometimes described by stating that the flow in the confining beds is essentially vertical and in the aquifer is essentially horizontal. Hantush's (9) analysis of an aquifer bounded only by one leaky confining bed suggested that this approximation is acceptably accurate wherever

$$\frac{K}{K'} > 100 \frac{b}{b'} \tag{8}$$

5.3.5 The Hantush-Jacob method requires that there is no change in water storage in the leaky confining bed. Weeks (10) states that if the "leaky" confining bed is thin and relatively permeable and incompressible, the solution of Hantush and Jacob (2) will apply, whereas the solution of Hantush (8), which is described in Test Method D 6028, that considers storage in confining beds will apply if at least one confining bed is thick, of low permeability, and highly compressible. For the case where one layer confining the aquifer is sensibly impermeable, and the other confining bed is leaky and bounded on the distal side by a layer in which the head is constant it follows from Hantush (8) that when time, t, satisfies

$$t > \frac{5(b')^2 S'_s}{K'} \tag{9}$$

the drawdowns in the aquifer will be described by the equation

$$s = \frac{Q}{4\pi T} W\left(u\delta, r \sqrt{\frac{K'}{Tb'}}\right) \tag{10}$$

where

$$\delta = 1 + \frac{S'}{3S} \tag{11}$$

Note that in Hantush's (8) solution, the term

$$u\delta = u\left(1 + \frac{S'}{3S}\right) = \frac{r^2 S}{4Tt}\left(1 + \frac{S'}{3S}\right) = \frac{r^2}{4Tt}\left(S + \frac{S'}{3}\right) \tag{12}$$

appears instead of the expression given for u in Eq 3, namely

$$u = \frac{r^2 S}{4Tt} \tag{13}$$

The implication being from Hantush (8) that after the time criterion given by Eq 9 is satisfied, the apparent storage coefficient of the aquifer will include the aquifer storage coefficient and one third of the storage coefficient for the confining bed. If the storage coefficient of the confining bed is very much less than that of the aquifer, then the effect of storage in the confining bed will be very small or sensibly nil. To illustrate the use of Hantush's time criterion, suppose a confining bed is characterized by $b' = 3$ m, $K' = 0.001$ m/day, and $S'_s = 3.6 \times 10^{-6}$ m^{-1}, then the Hantush-Jacob solution Eq

10 would apply everywhere when

$$t > \frac{5(b')^2 S'_s}{K'} = \frac{5(3\,\text{m})^2\,(3.6 \times 10^{-6}\,\text{m}^{-1})}{0.001\,(\text{m/day})} \quad (14)$$

or

$$t > 0.162\,\text{day} = 233\,\text{min} \quad (15)$$

If the vertical hydraulic conductivity of the confining bed was an order of magnitude larger, $K' = 0.01$ m/day, then the Hantush-Jacob (2) solution would apply when $t > 23$ min.

5.3.5.1 It should be noted that the Hantush (8) analysis assumes that well bore storage is negligible.

5.3.5.2 Moench (11) presents numerical results that give insight into the effects of control well storage and changes in storage in the confining bed on drawdowns in the aquifer for various parameter values. However, Moench does not offer an explicit formula for when those effects diminish enough for subsequent drawdown data to fit the Hantush-Jacob solution.

5.3.6 The assumption stated in 5.1.5, that the leaky confining bed is bounded on the other side by a uniform head source, the level of which does not change with time, was considered by Neuman and Witherspoon (12, p. 810). They considered a confined system of two aquifers separated by a confining bed as shown schematically in Fig. 3. Their analysis concluded that the drawdowns in an aquifer in response to discharging from a well in that aquifer would not be affected by the properties of the other, unpumped, aquifer for times that satisfy

$$t \le 0.1\,\frac{S'_s b'^2}{K'} \quad (16)$$

6. Apparatus

6.1 Analysis of data from the field procedure (see Test Method D 4050) by this test method requires that the control well and observation wells meet the requirements specified in the following paragraphs.

6.2 *Construction of Control Well*—Install the control well in the aquifer and equip with a pump capable of discharging water from the well at a constant rate for the duration of the test. Preferably, the control well should be open throughout the full thickness of the aquifer. If the control well partially penetrates the aquifer, take special precaution in the placement or design of observation wells.

6.3 *Construction and Location of Observation Wells and Piezometers*—Construct one or more observation wells or piezometers screened only in the pumped aquifer at a distance from the control well. Observation wells may be open through all or part of the thickness of the aquifer. Hantush (13, p. 350) indicates that the effects of a partially penetrating control well can be neglected for

$$r > 1.5b\,\sqrt{\frac{K_r}{K_z}} \quad (17)$$

where K_r and K_z are the aquifer hydraulic conductivities in the horizontal and vertical directions, respectively. If an observation well fully penetrates the aquifer, it's drawdown is not affected by a partially penetrating control well and it reacts as if the control well completely penetrated the aquifer (Hantush, 13, p. 351).

7. Procedure

7.1 Pretest preparations are described in detail in Test Method D 4050. The overall test procedure consists of (*1*) conducting the field procedure for withdrawal or injection well tests (described in Test Method D 4050) and (*2*) analysis of the field data, which is addressed in Section 8.

8. Calculation and Interpretation of Test Data

8.1 Aquifer-test data may be plotted in two ways (Cooper (4, p. C51)). Cooper (4) prepared two families of type curves that are plots of $L(u,v)$ versus $1/u$. Fig. 4 is a plot of a family
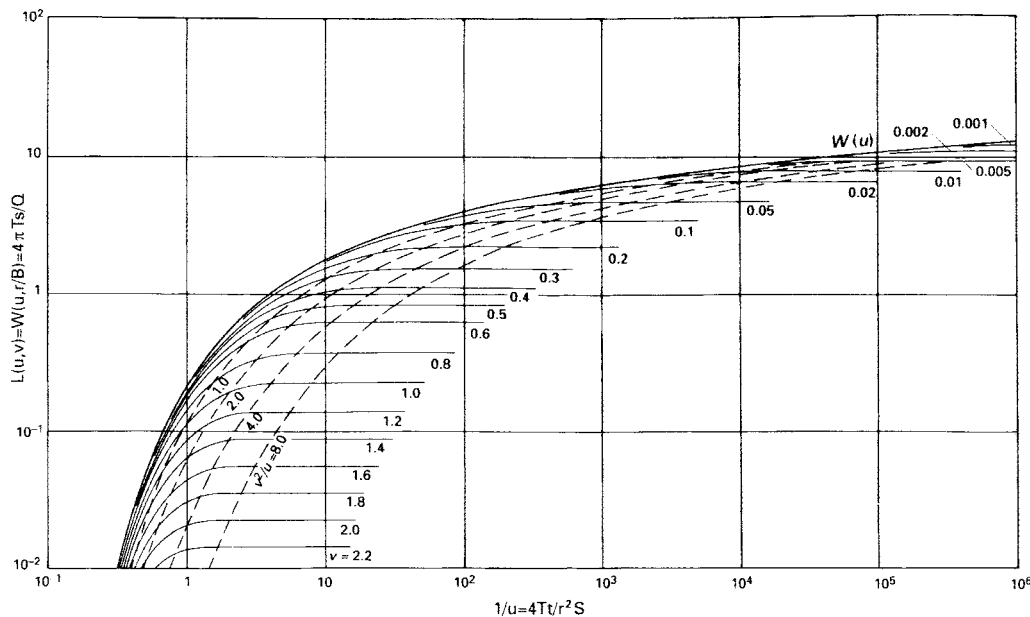

FIG. 4 Type Curve of L(u,v) versus 1/u (from Cooper (4)). The type curves for the region $v \le 1.2$ are based on data computed by H. H. Cooper, Jr., and Yvonne Clarke of the U.S. Geological Survey; those for the region $v \ge 1.4$ are based on data graphically interpolated from a table computed by Hantush ((3), p. 707–711)

of solid-type curves involving the parameter v (recall Eq 7, $v = \frac{r}{2}\sqrt{\frac{K'}{Tb'}}$) that are useful for a plot of drawdown versus time at some constant distance, r. For the other family of type curves, v^2/u (this is equal to $K't/Sb'$) there is the parameter for which type curves having different values are plotted (see Fig. 4, the dashed-line curves are the v^2/u curves). These curves are useful for a plot of drawdown versus $1/r^2$ at some constant time, t. Note that the parent curve of both families of curves is the Theis nonequilibrium type curve that corresponds to a nonleaky confined aquifer. Either family of type curves can be used to compute values of T, S, and K'/b'.

8.2 Except for a change in the notation used for the leakage coefficient, change of the equation numbers, and deletion of a small amount of text, the following description of the method of use of type curves is taken directly from Cooper (**4**, p. C51–C53).

8.2.1 To compute T, S, and K'/b' by use of the $v = \frac{r}{2}\sqrt{\frac{K'}{Tb'}}$ curves (solid-line type curves on Fig. 4), proceed as follows:

8.2.1.1 Plot s versus t/r^2 for each observation well on logarithmic graph paper having the same scale as the graph of the type curves.

8.2.1.2 Superpose this time-drawdown plot on the v curves and, keeping the coordinate axes of the two graphs parallel, translate the data plot to the position where the earliest data approach the limiting curve labeled $W(u)$ and all the remaining data for each well fall either between one pair of the curves labeled $v = 2.2$, $v = 2.0$, and so forth, or along one of them.

8.2.1.3 Select a convenient match point and note its coordinates (s, t/r^2, $L(u,v)$, and $1/u$).

8.2.1.4 Determine the value of v that corresponds to the value of r for each observation well. If the later data do not lie along one of the v-curves, estimate the value of v by interpolation.

8.2.1.5 Compute the hydraulic constants of the aquifer by making appropriate substitutions in the following equations:

$$T = \frac{Q}{4\pi}\frac{L(u,v)}{s} \tag{18}$$

$$S = 4T\frac{(t/r^2)}{1/u} \tag{19}$$

and

$$\frac{K'}{b'} = 4T\frac{v^2}{r^2} \tag{20}$$

8.2.2 To compute T, S, and K'/b' by use of $\frac{v^2}{u}$ curves (the dashed-line type curves on Fig. 4), proceed as follows:

8.2.2.1 Plot values of s, each from a different observation well but for identical values of t, versus t/r^2 on logarithmic graph paper having the same scale as the graph of the type curves.

8.2.2.2 Superpose this distance-drawdown plot on the v^2/u type curves and, keeping the coordinate axes of the two graphs parallel, translate the data plot to the position where all the data fall between one pair of the type curves or along one of them.

8.2.2.3 Select a convenient match point and note its coordinates (s, t/r^2, $L(u,v)$, and $1/u$).

8.2.2.4 Determine the value of v^2/u that corresponds to the value of t at which the drawdowns occurred. If the data do not lie along one of the type curves, estimate the value of v^2/u by interpolation.

8.2.2.5 Compute the values of T and S from Eq 17 and Eq 18. If the value is to be expressed in units consistent with those for T and S in Eq 17 and Eq 18, use

$$\frac{K'}{b'} = S\frac{v^2/u}{t} \tag{21}$$

If, when superposed on the v^2/u (dashed-line) type curves, the plotted data fall in the region $v^2/u \geq 8$ and $L(u,v) \geq 10^{-2}$, steady state conditions have been reached and the method of analysis suggested by Jacob (**14**) and described by Ferris et al (**15**, p. 112–115) is applicable.

8.2.3 Type curves of the Hantush-Jacob solution in the form developed by Cooper are available in numerous publications at scales convenient for matching against data plots. Some readily available sources of those type curves are Cooper (**4**), Lohman (**16**), and Reed (**1**). Cooper (**4**) illustrates the type curve procedure using hypothetical field data involving drawdowns at selected times for observation points at three different distances from a control well (see Note 2).

8.2.3.1 A procedure for analyzing data for steady-state conditions is described in 8.3.

8.2.3.2 Table 1 gives a tabulation of selected values of $W(u, r/B)$. If a set of type curves are not available these data can be used to develop a type curve plot. More detailed tabulations of the Hantush-Jacob solution are available from Hantush (**3,13**), and Walton (**17**). A computer program to compute values of $W(u,r/B)$ is on the diskettes that accompany Dawson and Istok (**18**). A listing for a computer program to compute values of $L(u,v)$ is in Reed (**1**) and a diskette including that program is available from the National Water Information System (NWIS) office of the U.S. Geological Survey.

8.2.3.3 Because the v curves represent different values of r/B, there is an advantage to having more than one observation well and for such wells to be at different distances from the control well so that a composite data-matching process can be used. Weeks (**10**) states of a composite data-curve matching process that:

Such a match should always be made when data from more than one observation well are available, and single values of transmissivity, storage coefficient, and other hydraulic properties are to be determined from that match. The ability or lack thereof of the data from observation wells at different distances to fit type curves having proportional distance-based parameters, will do much to confirm or deny the validity of the selected type-curve model. Moreover, the time-drawdown plot for any given observation well is affected by many extraneous factors, such as storage and inertial effects in the observation well, deviations of natural water-level fluctuations from those predicted from the pretest trend, barometric or loading effects on the water levels, and effects of local aquifer heterogeneity. Because most type-curve families include curves exhibiting a wide range of shapes, the chance of fortuitously fitting one of them is high when data for only a single well are matched. Thus, the composite data-curve matching process is useful both in confirming the validity of the selected model and in screening the data for extraneous effects.

NOTE 1—Spane and Wurstner (**19**) discussed the advantage of supplementing the type-curve plots of drawdown versus time by plots of the derivative of drawdown (with respect to an appropriate time function)

TABLE 1 Values of W(u,r/B) for Selected Values of u and r/B (from Hantush (13))

u	r/B							
	0.001	0.003	0.01	0.03	0.1	0.3	1	3
1×10^{-6}	13.0031	11.8153	9.4425	7.2471	4.8541	2.7449	0.8420	0.0695
2	12.4240	11.6716						
3	12.0581	11.5098	9.4425					
5	11.5795	11.2248	9.4413					
7	11.2570	10.9951	9.4361					
1×10^{-5}	10.9109	10.7228	9.4176					
2	10.2301	10.1332	9.2961	7.2471				
3	9.6288	9.7635	9.1499	7.2470				
5	9.3213	9.2618	8.8827	7.2450				
7	8.9863	8.9580	8.6625	7.2371				
1×10^{-4}	8.6308	8.6109	8.3983	7.2122				
2	7.9390	7.9290	7.8192	7.0685				
3	7.5340	7.5274	7.4534	6.9068	4.8541			
5	7.0237	7.0197	6.9750	6.6219	4.8530			
7	6.6876	6.6848	6.6527	6.3923	4.8478			
1×10^{-3}	6.3313	6.3293	6.3069	6.1202	4.8292			
2	5.6393	5.6383	5.6271	5.5314	4.7079	2.7449		
3	5.2348	5.2342	5.2267	5.1627	4.5622	2.7448		
5	4.7260	4.7256	4.7212	4.6829	4.2960	2.7428		
7	4.3916	4.3913	4.3882	4.3609	4.0771	2.7350		
1×10^{-2}	4.0379	4.0377	4.0356	4.0167	3.8150	2.7104		
2	3.3547	3.3546	3.3536	3.3444	3.2442	2.5688		
3	2.9591	2.9590	2.9584	2.9523	2.8873	2.4110	0.8420	
5	2.4679	2.4679	2.4675	2.4642	2.4271	2.1371	0.8409	
7	2.1508	2.1508	2.1506	2.1483	2.1232	1.9206	0.8360	
1×10^{-1}	1.8229	1.8229	1.8227	1.8213	1.8050	1.6704	0.8190	
2	1.2226	1.2226	1.2226	1.2220	1.2155	1.1602	0.7148	0.0695
3	0.9057	0.9057	0.9056	0.9053	0.9018	0.8713	0.6010	0.0694
5	0.5598	0.5598	0.5598	0.5596	0.5581	0.5453	0.4210	0.0681
7	0.3738	0.3738	0.3738	0.3737	0.3729	0.3663	0.2996	0.0639
1×10^{0}	0.2194	0.2194	0.2194	0.2193	0.2190	0.2161	0.1855	0.0534
2	0.0489	0.0489	0.0489	0.0489	0.0488	0.0485	0.0444	0.0210
3	0.0130	0.0130	0.0130	0.0130	0.0130	0.0130	0.0122	0.0071
5	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0008
7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001

versus time as an aide in selecting an aquifer interpretation model and in estimating the aquifer parameters. To apply the derivative methods requires that measurements be spaced closely enough that numerically developed time derivatives can be reasonably approximated.

8.2.4 Cooper (4) expressed some reservations about the use of this test method to determine values of the leakance, K'/b', for confining beds other than those that are sufficiently thin and for which the confining bed diffusivity K'/S'_s is sufficiently large. He noted that for confining beds that have a relatively large specific storage, much of the water yielded to the aquifer for a certain period of time would be derived from storage in the confining bed. For these reasons, the values of leakance obtained by this test method should be scrutinized considering independent geologic and hydrologic information.

NOTE 2—Following is an application of the type-curve method that Cooper (4) presented using "postulated" measurements of water-level drawdowns in observation wells at 100, 500, and 1000 ft from a well being pumped at a constant rate of 1000 gal/min for 1000 min. The aquifer in which the pumped well is screened is confined by a thin bed of materials whose lithologic character suggests that its ability to transmit water vertically through it may be an important factor that should be estimated. Fig. 5 is a log-log plot of the postulated drawdown data for each observation well plotted against values of t/r^2. Cooper's hypothetical "data" plot is superposed on a plot of the type curves (see Fig. 4) of $L(u,v)$ versus $1/u$.

8.2.4.1 For convenience, a match point is selected on the type-curve plot where $L(u,v) = 1.0$ and $1/u = 1.0$. For that choice, the corresponding point on the data plot gives $s = 1.15$

ft and t/r^2 of 1.87×10^{-9} (day/ft^2). Substitution of those values into Eq 17 and Eq 18 is as follows:

$$
\begin{aligned}
T &= \frac{Q}{4\pi} \frac{L(u,v)}{s} \\
&= \frac{1000 \text{ gal/min} \times (1440 \text{ min/day})}{4\pi \times 7.48 \text{ (gal/ft}^3)} \\
&\quad \times \frac{1.0}{1.15 \text{ ft}} \\
&= 13\,320 \text{ ft}^2/\text{day}
\end{aligned}
\tag{22}
$$

and

$$
S = 4T\frac{(t/r^2)}{1/u} = \frac{4(13\,320 \text{ ft}^2/\text{day}) (1.87 \times 10^{-9} \text{ day/ft}^2)}{1.0} = 0.0001
\tag{23}
$$

For the match position selected, Cooper estimated that the v curves that best fit the drawdown data are $v = 0.025$ for Observation Well 1 ($r = 100$ ft), $v = 0.125$ for Observation Well 2 ($r = 500$ ft), and $v = 0.25$ for Observation Well 3 ($r = 1000$ ft).

Considering the data at Observation Well 1, using Eq 19 the following is obtained:

$$
\frac{K'}{b'} = 4T\frac{v^2}{r^2} = 4(13\,320 \text{ ft}^2/\text{day}) \frac{(0.025)^2}{(100 \text{ ft})^2} = 3.3 (10^{-3}) \text{ day}^{-1}
\tag{24}
$$

The same value for K'/b' would be calculated for Observation Wells 2 and 3 because note that the ratios of v/r for Observation Wells 1, 2, and 3 turn out to be
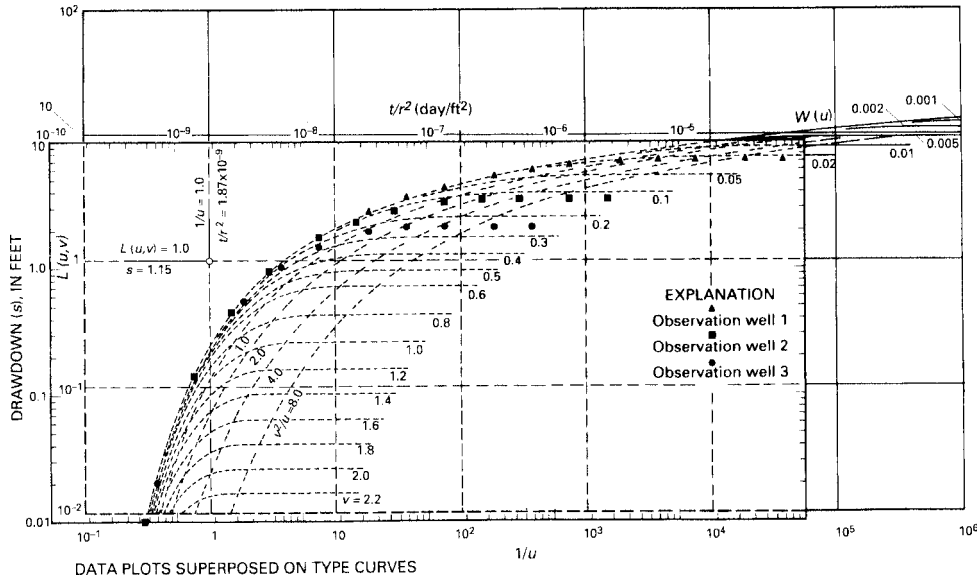
FIG. 5 Data Plot of Drawdown *s* versus Corresponding Values of *t/r²* (time/distance²) Superposed on the Type Curves Plot of *L(u,v)* versus 1/*u*

$$\frac{v}{r} = \frac{0.025}{100 \text{ ft}} = 0.00025 \text{ ft}^{-1} \qquad (25)$$

$$\frac{v}{r} = \frac{0.125}{500 \text{ ft}} = 0.00025 \text{ ft}^{-1}$$

$$\frac{v}{r} = \frac{0.25}{1000 \text{ ft}} = 0.00025 \text{ ft}^{-1}$$

respectively. This exact agreement reflects that the data used for the illustration are hypothetical and idealized. That the ratio of the *v*'s selected for the observation wells are in the same proportion as the distances to the observation wells is a desirable property to seek as Weeks **(10)** stresses because it is useful "in confirming the validity of the selected model and in screening the data for extraneous effects."

8.3 For the case where drawdowns in the vicinity of a control well have essentially reached a steady state, Jacob (**14**, p. 204) suggested a graphical type-curve method to analyze drawdowns at different distances to obtain an estimate of transmissivity and the coefficient of leakage. The steady-state drawdown is given by the following equation (Jacob, **14**, Eq 16):

$$s = \frac{Q}{2\pi T} K_0(x) \qquad (26)$$

where

$$x = \frac{r}{B} = r \sqrt{\frac{K'}{Tb'}} \qquad (27)$$

and where $K_0(x)$ is the zero-order modified Bessel function of the second kind.

8.3.1 The graphical type-curve procedure used to calculate aquifer test results is based on the functional relations between $K_0(x)$ and *s* and between *x* and *r*.

8.3.1.1 Plot values of $K_0(x)$ versus *x* on logarithmic paper (see Table 2 and Fig. 6). This plot is referred to as the type-curve plot. For convenience plot values of $K_0(x)$ on the vertical coordinate.

8.3.1.2 On logarithmic tracing paper of the same scale and size as the $K_0(x)$ versus *x* type curve, plot values of drawdown,

s, on the vertical coordinate versus the distance *r* on the horizontal coordinate.

8.3.1.3 Overlay the data plot on the type-curve plot and, while holding the coordinate axes of the two plots parallel, translate the data plot to a position that gives the best match to the type curve. Select and record the values of *x*, $K_0(x)$, *r*, and *s* at an arbitrary match point anywhere on the overlapping part of the two matched plots. It is convenient to select a match point where *x* and $K_0(x)$ are integers.

8.3.1.4 Using the coordinates of the arbitrarily selected point, the transmissivity and leakance are computed from Eq 21 and Eq 22:

$$T = \frac{Q}{2\pi s} K_0(x) \qquad (28)$$

$$\frac{K'}{b'} = \frac{x^2 T}{r^2} \qquad (29)$$

8.3.2 Hantush (**Eq 3**, see p. 703) notes that where $r/B \leq 0.05$ the Bessel function $K_0\left(\frac{r}{B}\right)$ is well approximated by a logarithmic function. Thus, in that range, a semilogarithmic plot of drawdown, *s*, versus distance, *r*, with *r* on the logarithmic scale, will give a straight-line relationship. The slope of that line, $\Delta s/\Delta \log_{10} r$, is equal to $(2.303Q)/(2\pi T)$, from which the transmissivity, *T*, can be calculated. This relationship indicates that in the region $r/B \leq 0.05$, the shape of the

TABLE 2 Values of the Bessel Function $K_0(x)$ for Selected Values of *x* (from Hantush (3, p. 704))

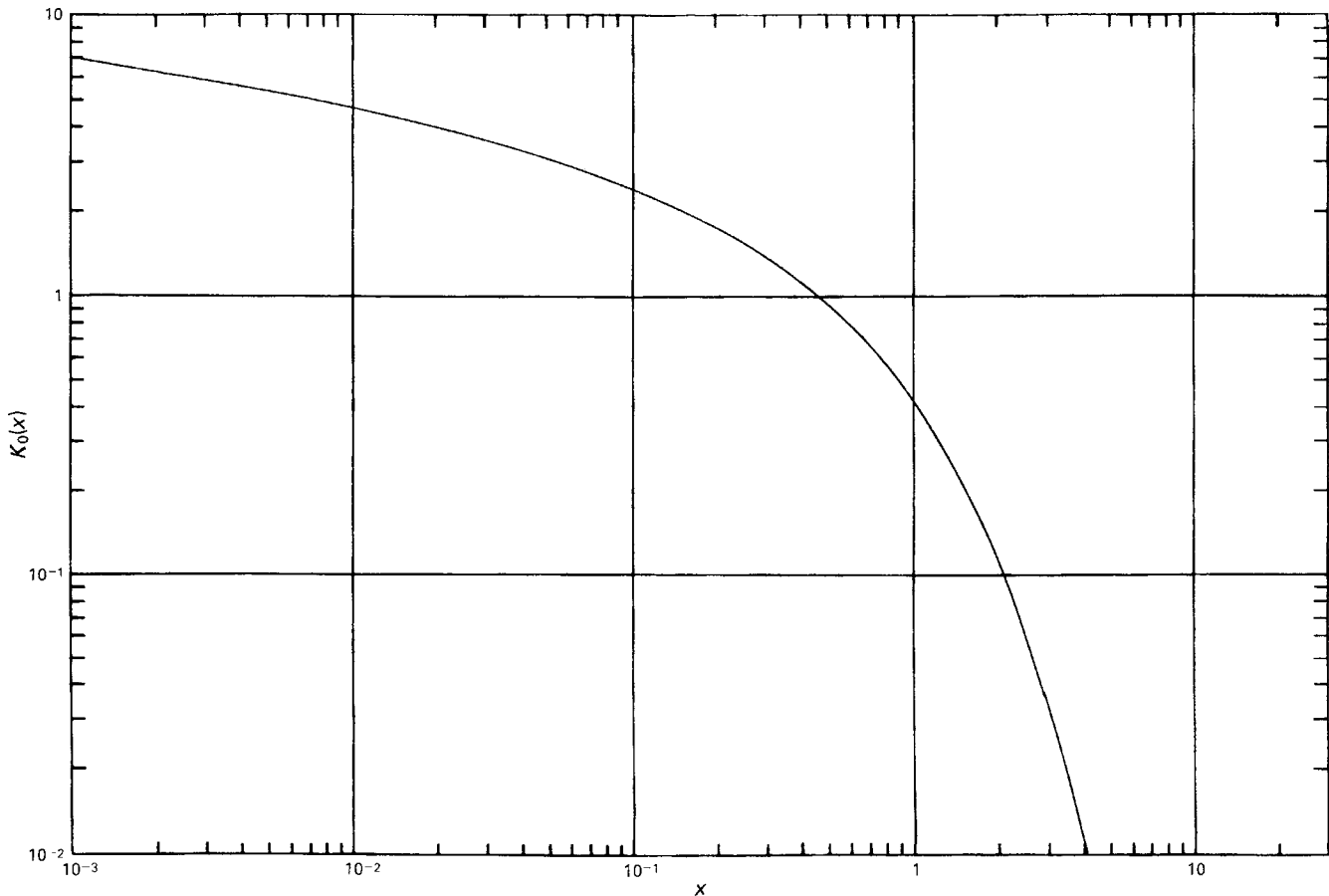| N	N × 10⁻²	10⁻¹	1
1	4.7212	2.4271	0.4210
1.5	4.3159	2.0300	0.2138
2	4.0285	1.7527	0.1139
3	3.6235	1.3725	0.0347
4	3.3365	1.1145	0.0112
5	3.1142	0.9244	0.0037
6	2.9329	0.7775	...
7	2.7798	0.6605	...
8	2.6475	0.5653	...
9	2.5310	0.4867	...

FIG. 6 Type Curve of the Bessel Function K_0, (x) as a Function of x (from Reed (1))

drawdown curve is not affected by the effects of leakage. Hantush also noted that the intercept, r_0, of this straight line at the zero-drawdown axis, has the property that $B = \sqrt{\dfrac{Tb'}{K'}}$ $= 0.89r_0$, from which the leakance $\dfrac{K'}{b'}$ can be calculated.

8.4 Qualitatively assess the test results considering the correspondence of the hydrogeologic conditions to the assumptions associated with the Hantush-Jacob (2) solution and the adequacy of the measurements of discharge and water-level changes.

9. Report

9.1 *Introduction*—The introductory section presents the scope and purpose of the Hantush-Jacob method. Summarize the field hydrogeologic conditions and the field equipment and instrumentation including the construction of the control well and observation wells and piezometers, or both, the method of measurement of discharge and water levels, and the duration of the test and pumping rate. Discuss the rationale for selecting the Hantush-Jacob formulation which assumes that although water moves through the confining bed(s) the gain or loss of water in storage in the confining bed(s) is negligible.

9.2 *Hydrogeologic Setting*—Review the information available on the hydrogeology of the site. Include the driller's logs and geologist's description of drill cuttings. Interpret and describe the hydrogeology of the site as it pertains to the

selection of the methods for conducting and analyzing an aquifer test. Compare the hydrogeologic characteristics of the site as it conforms and differs from the assumptions in the solution to the test method.

9.3 *Equipment*—Report the field installation and equipment for the aquifer test, including the construction, diameter, depth of screened interval, and location of control well and pumping equipment, and the construction, diameter, depth, and screened interval of observation wells or piezometers and their distances from the control well.

9.4 *Instrumentation*—Report the field instrumentation for observing water levels, pumping rate, barometric changes, and other environmental conditions pertinent to the test. Include a list of measuring devices used during the test; the manufacturer's name, model number, and basic specifications for each major item; and pertinent information on the method, including date, of the last calibration, if applicable.

9.5 *Testing Procedures*—State the steps taken in conducting pretest, drawdown, and recovery phases of the test. Include the frequency of measurements of discharge rate, water level in observation wells, and other environmental data recorded during the test procedure.

9.6 *Presentation and Interpretation of Test Results*:

9.6.1 *Data*—Present tables (and charts for graphically recorded data) of data collected during the test (pretest and recovery included). Show methods of adjusting water levels for

barometric changes, tidal changes, or other background water level changes (interference with other operations and boundary conditions) and calculation of drawdown.

9.6.2 *Data Plots*—Present data plots used in analysis of the data. Show overlays of data plots and type curves with match points and corresponding values of parameters at match points.

9.6.3 *Calculations*—Show calculations of transmissivity, storage coefficient, and coefficient of leakage.

9.7 Evaluate qualitatively the overall accuracy of the test method on the basis of the adequacy of instrumentation and observations of stress and response, and the conformance of site assumptions to test results.

10. Precision and Bias

10.1 It is not practical to specify the precision of this procedure in this test method because the response of aquifer systems during aquifer tests is dependent upon ambient system stresses. No statement can be made about bias because no true reference values exist.

11. Keywords

11.1 aquifers; aquifer tests; confined aquifers; confining beds; control wells; ground water; hydraulic properties; leakance; leaky aquifers; observation wells; storage coefficient; transmissivity

REFERENCES

(**1**) Reed, J. E., "Type Curves for Selected Problems of Flow to Wells in Confined Aquifers," *U.S. Geological Survey Techniques of Water-Resources Investigation Book 3*, Chapter B3, 1980.

(**2**) Hantush, M. S., and Jacob, C. E., "Non-Steady Radial Flow in an Infinite Leaky Aquifer," *Transactions, American Geophysical Union*, Vol 36, No. 1, 1955, pp. 95–100.

(**3**) Hantush, M. S., "Analysis of Data From Pumping Tests in Leaky Aquifers," *Transactions, American Geophysical Union*, Vol 37, No. 6, 1956, pp. 702–714.

(**4**) Cooper, H. H., Jr., "Type Curves for Nonsteady Radial Flow in an Infinite Leaky Artesian Aquifer," in *Shortcuts and Special Problems in Aquifer Tests*, Ray Bentall, Compiler, U.S. Geological Survey Water-Supply Paper 1545-C, 1963, pp. C48–C55.

(**5**) Abdul Khader, M. H., and Ramadurgaiah, D., "Flow Towards a Well of Large Diameter in a Leaky Confined Aquifer," *6th Australasian Hydraulics and Fluid Mechanics Conference*, Adelaide, Australia, December 5–9, 1977, pp. 84–87.

(**6**) Papadopulos, I. S., and Cooper, H. H., Jr.," Drawdown in a Well of Large Diameter," *Water Resources Research*, Vol 3, No. 1, 1967, pp. 241–244.

(**7**) Neuman, S. P., and Witherspoon, P. A., "Field Determination of the Hydraulic Properties of Leaky Multiple Aquifer Systems," *Water Resources Research*, Vol 8, No. 5, 1972, pp. 1284–1298.

(**8**) Hantush, M. S., "Modification of the Theory of Leaky Aquifers," *Journal of Geophysical Research*, Vol 65, No. 11, 1960, pp. 3713–3725.

(**9**) Hantush, M. S., "Flow of Ground Water in Relatively Thick Leaky Aquifers," *Water Resources Research*, Vol 3, No. 2, 1967, pp. 583–590.

(**10**) Weeks, E. P., "Aquifer Tests—The State of the Art in Hydrology," *Proceedings of the Invitational Well-Testing Symposium*, Berkeley, California, October 19–21, 1977, LBL7027, Lawrence Berkeley Laboratory, pp. 14–26.

(**11**) Moench, A. F., "Transient Flow to a Large-Diameter Well in an Aquifer With Storative Semiconfining Layers," *Water Resources Research*, Vol 21, No. 8, pp. 1121–1131.

(**12**) Neuman, S. P., Witherspoon, P. A., "Theory of Flow in a Confined Two Aquifer System," *Water Resources Research*, Vol 5, No. 4, 1969, pp. 803–816.

(**13**) Hantush, M. S., "Hydraulics of Wells," in Chow, Ven Te, Ed., *Advances in Hydroscience*, Vol 1, New York, Academic Press, 1964, pp. 281–432.

(**14**) Jacob, C. E., "Radial Flow in a Leaky Artesian Aquifer," *Transactions American Geophysics Union*, Vol 27, No. 2, 1946, pp. 198–205.

(**15**) Ferris, J. G., Knowles, D. B., Brown, R. H., and Stallman, R. W.," Theory of Aquifer Tests," *U.S. Geological Survey Water-Supply Paper* 1536-E, 1962.

(**16**) Lohman, S. W.," Ground-Water Hydraulics," *U.S. Geological Survey Professional Paper 708*, 1972.

(**17**) Walton, W. C., "Selected Analytical Methods for Well and Aquifer Evaluation," *Illinois State Water Survey Bulletin 49*, 1962.

(**18**) Dawson, K. J., and Istok, J. D., "Aquifer Testing, Design and Analysis of Pumping and Slug Tests," Lewis Publishers, Inc., 1991.

(**19**) Spane, F. A., Jr., and Wurstner, S. K., "DERIV: A Computer Program for Calculating Pressure Derivatives for Use in Hydraulic Test Analysis," *Ground Water*, Vol 31, No. 5, 1993, pp. 814–822.