

# Measurement of liquid flow in open channels —

Part 2: Dilution methods —

Part 2A: General

# Committees responsible for this British Standard

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Clyde River Purification Board  
 Department of the Environment  
 Institute of Measurement and Control  
 Institution of Water and Environmental Management  
 National Rivers Authority  
 Water Services Association of England and Wales  
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Institute of Hydrology  
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# Contents

	Page
Committees responsible	Inside front cover
National foreword	ii
<hr/>	
Introduction	1
1 Scope	1
2 Normative reference	1
3 Definitions	1
4 Symbols and units	4
5 Principles of the different dilution methods	5
6 Characteristics common to both methods	6
7 Choice of tracer	11
8 Choice of measuring reach	12
9 Procedure for the constant rate injection method	20
10 Procedure for the sudden injection method	25
11 Comparison between the two dilution methods	30
12 Estimation of uncertainties	32
<hr/>	
Annex A (normative) Standard dilutions using pipettes and flasks	46
Annex B (normative) Standard dilution ranges obtained with a micrometer-type microburette	47
Annex C (normative) Analysis for variance as a test of non-randomness	49
Annex D (informative) Bibliography	50
<hr/>	
Figure 1 — Relation between $\varepsilon$ and $c_2$	12
Figure 2 — Curves of appearance and disappearance of tracer for a sudden injection — Shapes of curves of concentration against time and distance	18
Figure 3 — Curves of appearance and disappearance of tracer used to determine a suitable duration of injection $T_i$ for a gauging by the constant rate injection method — Shapes of curves of concentration against time and distance	19
Figure 4 — Constant level tank for the injection of tracer solution at constant rate	22
Figure 5 — Use of a constant-rate pump for injection of tracer solution at constant rate	23
Figure 6 — Mariotte constant head vessel	24
Figure 7 — Floating siphon device	24
Figure 8 — Typical sampling curves	27
Figure 9 — Obtaining mean samples	28
Figure 10 — Obtaining several mean samples	31
Figure 11 — Pump with manifold for multiple sampling	32
<hr/>	
Table 1 — Values of the coefficient $k$	15
Table B.1 — Preparation of standard dilution ranges using a micrometer-type microburette	48
Table B.2 — Micrometer turns for ranges with mid-values $N_0$	48
Table C.1 — Instrument readings	49
Table C.2 — Sums of squares of values from Table C.1	49
Table C.3 — Instrument readings	50
Table C.4 — Sum of squares of values from Table C.3	50
<hr/>	
List of references	Inside back cover
<hr/>	

# National foreword

This Part of BS 3680 has been prepared under the direction of the Electrotechnical Sector Board. It is identical with ISO 9555-1:1994 *Measurement of liquid flow in open channels — Tracer dilution methods for the measurement of steady flow — Part 1: General*, published by the International Organization for Standardization (ISO).

This is one of a series of Parts of BS 3680 on dilution methods, as follows:

- *Part 2A:1994: General*;
- *Part 2B:1993: Radioactive tracers*;
- *Part 2C:1993: Chemical tracers*;
- *Part 2D:1993: Fluorescent tracers*.

These four Parts of BS 3680 supersede BS 3680-2A:1964, BS 3680-2B:1986, and BS 3680-2C:1983 which are withdrawn.

## Cross-references

The Technical Committee has reviewed the provisions of ISO 772:1988, to which normative reference is made in the text, and has decided that they are acceptable for use in conjunction with this standard.

Informative reference is also made to the following standard:

ISO 8363:1986	BS 3680 <i>Measurement of liquid flow in open channels</i> Part 3G:1990 <i>General guidelines for the selection of methods</i>
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**Compliance with a British Standard does not of itself confer immunity from legal obligations.**

## Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii, pages 1 to 50, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

## Introduction

The former standard series ISO 555 was subdivided into parts on the basis of the method of field measurement, i.e. constant-rate injection method and integration (sudden injection) method. Since the choice of the type of tracer to be used in a field measurement will often depend on the expertise and the laboratory facilities available, this new series of standards ISO 9555 is divided into parts based on the type of tracer used. This revision has enabled the unnecessary repetition of text of the various parts to be avoided and will, it is hoped, prove to be a more convenient form of presentation for the user.

ISO 9555 deals with the measurement of steady flow in open channels by dilution methods using tracers. The methods described may also be applied to the measurement of slowly varying flow, but they may only be used when flow conditions ensure adequate mixing of the injected solution throughout the flow.

For the measurement of very large flows, tracer methods can be onerous in terms of tracer costs and measurement times. However, the use of tracers often reduces danger to personnel during flood periods.

ISO 9555-1 presents the general principles of the methods of constant rate injection and integration (sudden injection). ISO 9555-2, ISO 9555-3 and ISO 9555-4 deal with the specific aspects of the use of radioactive, chemical and fluorescent tracers, respectively, as well as specific analytical procedures.

This approach has been adopted for the following reasons:

- to facilitate subsequent updating, additions or revisions which concern only ISO 9555-2, ISO 9555-3 or ISO 9555-4;
- to provide a more practical document for the user, who is often obliged to choose the tracer best suited to the available analytical equipment.

## 1 Scope

This part of ISO 9555 concerns principles of the constant rate injection method and the sudden injection method (integration method), and the principles of injection, sampling and analysis which apply for all types of tracer.

It is important to note that the tracers used and their concentrations should be compatible with the legislation of each country regarding the occasional and short-duration discharges of foreign substances into natural waters. They should also take into account possible short-term or long-term influences on flora and fauna, both natural and domestic. In the absence of appropriate regulations, the World Health Organization's (WHO) *Guidelines for drinking water quality* should serve as a guide, taking account of the brief nature of the measurement and of the use that is to be made of water receiving the injected tracer.

## 2 Normative reference

The following standard contains provisions which, through reference in this text, constitute provisions of this part of ISO 9555. At the time of publication, the edition indicated was valid. All standards are subject to revision, and parties to agreements based on this part of ISO 9555 are encouraged to investigate the possibility of applying the most recent edition of the standard indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 772:1988, *Liquid flow measurement in open channels — Vocabulary and symbols*.

## 3 Definitions

For the purposes of this part of ISO 9555, the definitions given in ISO 772 and the following definitions apply.

### 3.1 tracer

ion, compound or radionuclide introduced into a flow system to follow the behaviour of some component of that system. It is necessary that the tracer, which can be observed, behave in exactly the same fashion as the component to be followed, whose behaviour cannot easily be observed

### 3.2 dilution method

method in which the discharge is deduced from the determination of the ratio of the concentration of the tracer injected to that of the tracer at the sampling cross-section (see also 3.9)

**3.3**

**constant rate injection method**

method of measuring the discharge in which a tracer is injected at a constant and known rate at one cross-section and its dilution is measured at another section downstream where complete mixing has taken place and the concentration plateau has been reached

**3.4**

**sudden injection method; integration method**

method of measuring the discharge in which a known volume of tracer is injected over a short time at one cross-section and its dilution is measured at another cross-section downstream where complete mixing has taken place, over a period sufficient to allow all the tracer to pass that cross-section, so that the mean dilution of tracer during this period can be determined

**3.5**

**multiple injection**

simultaneous injection of tracer at several points in the injection cross-section, with the aim of improving transverse mixing in a given measuring reach

**3.6**

**constant level tank**

equipment for the injection of the concentrated solution at constant rate, in which, using an overflow weir, a constant head is maintained above a selected nozzle or orifice

**3.7**

**Mariotte vessel**

equipment for injection of the concentrated solution at constant rate. The constant rate injection is achieved by means of an airtight vessel that is provided with an orifice plate or nozzle at its bottom portion. The liquid flows through the restriction and air enters the vessel through a tube, maintaining atmospheric pressure at the lower end of the tube, set at a determined height above the restriction. The head on the restriction and consequently the discharge from the vessel remain constant, independently of the level of the liquid in the vessel

**3.8**

**floating siphon**

equipment for injection of the concentrated solution at constant rate. The solution is taken from a vessel by means of a siphon that is fixed on a float. The lower end of the siphon is fitted with an orifice plate or nozzle. The head on the restriction and consequently the discharge from the vessel remain constant, independently of the level of the liquid in the vessel

**3.9**

**dilution ratio; dilution factor**

- 1) For the constant rate injection method, the ratio between the concentration of the injected tracer solution and the concentration of added tracer detected at the sampling cross-section when steady conditions have been reached.
- 2) For the sudden injection method, the ratio between the concentration of the injected tracer solution and the mean, over the sampling period, of the concentration of added tracer detected at the sampling cross-section.

**3.10**

**sampling cross-section; sampling station**

cross-section of an open channel downstream of the injection cross-section, at which samples are taken or in which the concentration is directly measured

**3.11**

**reach**

length of open channel between two defined cross-sections

**3.12****degree of mixing**

measure of the extent to which mixing has been achieved in a cross-section downstream of the injection of tracer, varying from near zero in a cross-section immediately downstream of the injection to 100 % at a cross-section in which the tracer has been completely mixed across the whole cross-section

**3.13****mixing length**

distance, measured along the general path of flow, between the injection cross-section and the downstream cross-section at which the specified degree of mixing is obtained. For given conditions, this mixing length is not a fixed value: it varies according to the admissible values for the degree of mixing. The higher the specified degree of mixing, the longer the mixing length

## 4 Symbols and units

### 4.1 Symbols

The symbols used in this part of ISO 9555 are given below. Other symbols not listed here are used only in passing and are defined where used.

Symbol	Explanation
$\bar{b}$	Average stream surface width between the injection and sampling cross-sections
$c_0$	Background concentration of tracer
$c_1$	Concentration of tracer in injection solution
$c_2$	Concentration of added tracer
$c'_1$	Concentration of diluted injection solution ( $c'_1 = c_1/D'$ )
$c'_{ri}$	Relative concentration corresponding to standard dilution $D'$
$D$	Dilution factor
$D'$	Estimated dilution factor used in dilution of injection solution
$D'_i$	Standard dilution
$\bar{d}$	Average depth of flow between injection and sampling cross-sections
$E$	Transverse mixing coefficient
$G$	Instrument response
$g$	Acceleration due to gravity
$L$	Mixing length
$M, m$	Mass
$m$	Number of sampling points
$n$	Number of samples or measurements
$Q$	Stream discharge
$Q'$	Estimated stream discharge
$q$	Injection rate
$r_h$	Mean hydraulic radius
$S$	Energy slope
$T_c$	Duration of steady state concentration
$T_i$	Duration of injection
$T_p$	Time of passage of tracer through the sampling cross-section
$t$	Elapsed time, usually taking as the origin the start of injection
$t_a$	Time of arrival of first molecule of tracer at a given sampling cross-section
$t^*$	Mean time of travel
$V$	Volume of tracer solution injected
$\bar{v}$	Mean velocity
$v^*$	Shear velocity, equal to $(gr_h S)^{1/2}$
$x$	Degree of mixing
$\epsilon$	Level of accuracy of analysis
$\theta$	Lowest temperature likely to be met during gauging

### 4.2 Subscripts

c	Concentration
z	Exponent

NOTE 1 A bar (–) is used above the symbol to indicate an averaging process; further averaging is indicated by a double bar (=).

NOTE 2 For convenience in the use of this part of ISO 9555, some of the symbols and their definitions are not consistent with those defined in ISO 772.

### 4.3 Units of measurement

The units of measurement used in this part of ISO 9555 are those of the International System of Units (SI).

## 5 Principles of the different dilution methods

The basic principle of dilution methods is the addition of a suitably selected tracer to the flow. Downstream of the injection point, when vertical and lateral dispersion throughout the flow is complete, the discharge may be calculated from the measurement of the dilution of the tracer.

If this tracer, or a substance with analytical characteristics similar to the injected tracer, was present in the flow before the injection, the increase in concentration of tracer due to the injection is known as the "concentration of added tracer". The methods set out in this part of ISO 9555 permit the calculation of the discharge regardless of the prior concentration of tracer, analogous with background noise, as long as this can be assumed constant during the measurement.

### 5.1 Constant rate injection method

A solution of concentration  $c_1$  of a suitably chosen tracer is injected at a constant rate  $q$  into a cross-section located at the beginning of the measuring reach of the channel, in which the discharge  $Q$  remains constant for the duration of the gauging.

At a second cross-section downstream of this reach, at a sufficient distance for the injected solution to be uniformly diluted, the concentration is measured for a sufficient period of time and at a sufficient number of points to ensure that good mixing has been obtained and that the concentration of added tracer  $c_2$  has attained a constant value. Under these conditions, if all of the tracer injected passes through the sampling cross-section, the discharge rate of tracer at the injection points is equal to that passing through the sampling cross-section:

$$qc_1 = (Q + q)c_2$$

Hence:

$$Q = q \frac{c_1 - c_2}{c_2} \quad \dots(1)$$

In general  $c_1$  is much greater than  $c_2$  and this leads to a simplification of equation (1) as follows:

$$Q = q \frac{c_1}{c_2} \quad \dots(2)$$

The discharge  $Q$  can therefore be determined by comparing the concentration of the injection solution with that measured at the sampling cross-section of the channel.

### 5.2 Sudden injection method

A volume  $V$  of a solution of concentration  $c_1$  of a suitably chosen tracer is injected over a short period into a cross-section located at the beginning of the measuring reach of the channel, in which the discharge  $Q$  remains constant for the duration of the gauging.

At a second cross-section downstream of this reach, at a distance sufficient for the injected solution to be uniformly diluted, the concentration of tracer  $c_2$  is determined over a period of time sufficiently long to ensure that all the tracer has passed through the second cross-section.

If all the tracer injected passes through the sampling cross-section, the following equation is true:

$$M = Vc_1 = Q \int_{t_1}^{\infty} c_2(t) dt \quad \dots(3)$$

where

- $M$  is the mass of tracer injected;
- $V$  is the volume of injected solution;
- $c_1$  is the concentration of tracer in the injected solution;
- $Q$  is the discharge in the channel;

$c_2(t)$  is the concentration of added tracer at the fixed sampling point over the time interval  $d_t$ ;

$t$  is the elapsed time, taking as origin the instant at which the injection started;

$t_a$  is the time of arrival of the first molecule of tracer at the sampling cross-section.

This equation requires that the value of the integral

$$\int_{t_a}^{\infty} c_2(t) dt$$

be the same at every point of the sampling cross-section. This condition is satisfied if and only if the injected solution is well mixed with the water in the channel.

In practice, the presence of the tracer is no longer detectable at any point in the sampling cross-section after a certain time ( $t_a + T_p$ ). The value  $T_p$  is known as the "time of passage of the tracer cloud through the sampling cross-section".

Let

$$\bar{c}_2 = \frac{1}{T_p} \int_{t_a}^{t_a + T_p} c_2(t) dt$$

The practical condition of good mixing is that  $\bar{c}_2$  is identical at all points of the section, hence

$$Q = \frac{Vc_1}{T_p \bar{c}_2} \quad \dots(4)$$

$V/T_p$  is the mean discharge of added tracer in the sampling cross-section, and is analogous with the injection rate  $q$  defined in 5.1.

## 6 Characteristics common to both methods

### 6.1 General formula for the calculation of discharge

Relations (2) and (4) can be written in the general form:

$$Q = kD \quad \dots(5)$$

For measurement by constant rate injection,  $D$  is the dilution factor ( $= c_1/c_2$ ) and  $k = q$ .

For measurement by sudden injection,  $D = c_1/\bar{c}_2$  and  $k = V/T_p$ .

The dilution factor,  $D$ , which is the fundamental concept of the dilution method, is determined by comparative analysis of samples of the injection solution and samples taken from the sampling cross-section.

This determination, although certain aspects are peculiar to the tracer and to the associated method of analysis, rests on several general principles which are presented below. The details of the specific methods of determination for the commonly-used tracers are given in ISO 9555-2, ISO 9555-3 and ISO 9555-4.

### 6.2 Separate determination of tracer concentrations in the channel and in the injection solution

The practice of dilution gauging shows that in most cases the pursuit of maximum effectiveness and convenience leads to a reduction in the total quantities injected, and to a dilution factor,  $D$ , which attains high values, of the order of  $10^3$  to  $10^8$ .

All analytical procedures, whatever the tracer, the apparatus or the method of analysis, are based on the existence of a physical relation between the concentration of tracer and a measurable quantity  $G = G(c)$ , for which the analytical instrument provides a value  $G_i$  corresponding to  $c_i$ . Experience shows that it is difficult, if not impossible, directly to obtain comparable accuracy for independent measurements of concentrations varying in so high a ratio as  $D$  with the same instrument or method of analysis.

#### 6.2.1 Direct determination of $c_1$ and $\bar{c}_2$

Procedures for the direct determination of  $c_1$  and  $\bar{c}_2$  are not recommended and thus are not discussed in this part of ISO 9555.

### 6.2.2 Indirect determination of $c_1$ and direct determination of $\bar{c}_2$

Where the analytical method leads to an instrument response which is a linear function of concentration in the range from zero to the usual value of  $c_2$ , the following procedure of indirect determination of  $c_1$  can be used. The solution of concentration  $c_1$  is diluted by a factor  $D'$ , of the same order as  $D$ , to obtain concentration  $c'_1 = c_1/D'$  and  $c_2$  of the same order of magnitude. This permits use of the same method for the determination of  $c'_1$  and  $c_2$  with comparable accuracy. For this purpose, a sample of injection solution is diluted by the gravimetric method using a precision balance, or by the volumetric method using flasks, calibrated pipettes or microburettes, preferably with water taken from the flow upstream of the injection cross-section or before the test.

This method of determination is a simplified variant of the procedure described in 6.3.

### 6.3 Direct determination of the dilution factor

The method of comparative or standard dilutions is particularly recommended because it minimizes the effects of systematic errors in the determination.

#### 6.3.1 Principle

After estimating  $D'$  from the dilution factor  $D$  [using formula (5)] from the test conditions and from the estimate  $Q$ , however rough, of the discharge  $Q$  to be measured, the method consists of preparing a series of standard dilutions of a sample of the injected solution, having known dilution factors  $D'_1, D'_2, \dots, D'_n$  surrounding the estimated value  $D'$ .

The analytical equipment is then used as a comparator to determine the value of the dilution factor  $D$ . The basic requirements of the equipment are thus sensitivity and stability. The latter is necessary only during the analysis of samples corresponding to a single flow measurement.

Experience shows that the ratio  $D'/D$  rarely falls outside the range 0,5 to 2. It is therefore possible to use the analytical equipment in the vicinity of its optimum range of sensitivity and accuracy.

Let  $G'_1, G'_2, \dots, G'_n$  be the responses of the analytical equipment for standard dilution factors  $D'_1, D'_2, \dots, D'_n$  and let  $G_1, G_2, \dots, G_n$  be the responses of the analytical equipment for given river samples corresponding to unknown dilutions  $D_1, D_2, \dots, D_n$ .

It is then necessary to obtain the dilutions  $D_i$  corresponding to the responses  $G_i$  by using the relation between  $D'$  and  $G'$  established for the analytical equipment.

This relationship is established either graphically or by using a mathematical function.

As the response of analytical instruments is generally a linear function of concentration, it is logical to use the concept of relative concentration  $c'_{ri} = 1/D'_i$ .

The graphical representation of the relation  $G' = G'(c'_r)$  is a simple means of checking its linearity and the accuracy of preparation of the standard dilutions.

#### 6.3.2 Application to measurement by the constant rate injection method

When the gauging is performed in good conditions, the scatter of the  $G_i$  about their mean  $G$  is small, and a small number of standard dilutions suffices to define the response function over the range used ( $D'/2$  to  $2D'$ ).

#### 6.3.3 Application to measurement by the sudden injection method

The necessary range of  $G'_i$  varies according to the method of sampling selected (see 10.4.2). In the case of a mean sample (see 10.4.2.3) or several mean samples (see 10.4.2.4), conditions differ little from those of the constant rate injection method.

In the case of discrete samples (see 10.4.2.2) or continuous recording, the concentration  $c_{2i}$  of the samples may vary over a very large range, and it is necessary to determine the response of the analytical equipment by a larger number of standard dilutions distributed over an interval which could extend for example from  $D'/10$  to  $10D'$ .

#### 6.3.4 Preparation of standard dilutions

##### 6.3.4.1 Definition

The set of standard dilutions required is defined by the number of standard dilutions, their range and their distribution around the central value.

The range is a function of the accuracy with which field conditions permit the estimation of the dilution factor  $D'$ , which corresponds to the central value<sup>1)</sup>. The set of dilutions will comprise five to seven values, in a sequence approximating a geometric sequence.

For example, using either dilution  $D_i$  ( $= 1/c_{ri}$ ) or relative concentration ( $c_{ri}$ ), most purposes can be served by selecting seven consecutive values from the following reference series, multiplied by a factor of  $10^z$ . The range can be extended by choosing alternate values.

For relative concentrations:

0,8; 1; 1,25; 1,5; 2; 2,5; 3; 4; 5; 6; 8; 10; 12,5

For dilutions:

12,5; 10; 8; 6,4; 5; 4; 3,2; 2,5; 2; 1,6; 1,25; 1; 0,8

The establishment of standard ranges of this type, associated with a detailed laboratory procedure, allows significant savings of time in preparation and analysis in the laboratory or in the field, gives measurements which are more comparable, and aids in the choice of the most appropriate equipment for the smallest suitable number of dilutions.

Two methods can be used to obtain the sets of standard dilutions: the volumetric method and the gravimetric method.

#### 6.3.4.2 Volumetric method

This method uses calibrated glassware (i.e. pipettes, flasks and microburettes) to prepare the required series of standards by successive dilution operations. Essential precautions for obtaining suitable accuracy by this method are as follows:

- select, from the apparatus conforming to the relevant standards, that which is most convenient to use and provides an accuracy appropriate to the problem;
- avoid in particular pipettes of volume less than 10 ml and flasks of volume less than 250 ml. One-mark bulb-type pipettes, which are most easily handled, are preferred;
- follow scrupulously the recommendations for use laid down in the relevant standards;
- maintain all glassware in a perfect state of cleanliness and discard any equipment which is chipped or stained.

##### 6.3.4.2.1 Use of pipettes and flasks only

This is the least sophisticated and the least expensive procedure, but it is not always possible to obtain conveniently the values proposed for the reference series. A guide to the procedure and a list of recommended equipment is presented in Annex A.

To obtain the best results, the following additional recommendations are useful:

- one series of pipettes and flasks should be clearly marked and reserved for the initial dilution operations which involve the highest concentrations and the greatest risks of persistent contamination, despite washing;
- each pipette should be rinsed several times with the solution which it is to deliver;
- it is recommended that tests of the effectiveness of cleaning procedures be performed regularly, for example by rinsing supposedly clean equipment with distilled water and examining for tracer in the rinse water;
- the work plan of the laboratory should be rigorously organized to separate the zones of preparation of samples, for first dilutions of intermediate concentration and for standard dilutions, to avoid accidental contamination.

Disadvantages of this method are

- the inconvenience of its use in the field;
- in the case of large dilution factors, the number of successive dilutions necessary, which prolongs the operation and leads to a possible loss of accuracy.

The advantage of this method is that the equipment can be easily obtained at moderate cost, even for equipment of the highest accuracy.

<sup>1)</sup> This range is also dependent on the method used (see 6.3.2 and 6.3.3)

Although the methods set out in 6.3.4.2.2 and 6.3.4.3 are preferred, it is desirable to have the necessary equipment for the application of this method in emergencies.

#### 6.3.4.2.2 Microburettes

Microburettes (also known as microsyringes) consist of a cylinder containing a piston which is driven by a manually controlled micrometer device (hand-operated type) or by an electromechanical device; some types permit the delivery of volumes adjustable continuously from 0,01 ml to 0,5 ml with an accuracy better than 0,5 %. Thereby it is possible to obtain very conveniently suitable ranges of standards distributed around the reference dilution  $D'$ , by injecting directly into a series of beakers containing for example 50 ml of river water. Under these conditions and depending on the final accuracy required, it may be necessary to carry out a strength correction of the dilution due to the addition of the volume  $V$  delivered by the microburette into the reference volume,  $V_{\text{ref}}$ , of the standard dilution,  $[V/(V_{\text{ref}} + V)]$  instead of  $V/V_{\text{ref}}$ .

It is possible, using for the first dilution either a pipette and a flask or another microburette, to obtain dilution factors of  $10^6$  or  $10^7$  with only two successive dilution operations.

The advantages of microburettes are

- a reduction in the amount of glassware necessary, in the length of the dilution operations and in the quantities of background water necessary for the standard dilutions;
- a consistent accuracy, practically independent of the dilution factor. Moreover, their compact size permits their use in the field.

As in the use of the pipette-flask procedure, it is possible to define predetermined ranges of standards, in order to distribute the dilutions evenly around the estimated value, avoiding the loss of time caused by calculation for each analysis and limiting the necessary glassware (see Annex B).

#### 6.3.4.3 Gravimetric method

This method uses similar glassware, but does not rely on its precise calibration, the dilution factor of each successive operation being determined by weighing on a precision balance. The sequence of operations is described below:

- a) Determine the volume  $V$  of the required diluted solution and the volume  $V_1$  of the solution to be diluted considering the desired dilution factor  $D = V_1/V$ .
- b) Fill a small vessel, e.g. an ampoule or syringe, of suitable capacity with a volume approximately equal to  $V_1$  of the solution to be diluted, seal the ampoule and weigh it: its mass is  $m_2$ .
- c) Weigh an empty flask of volume  $V_0$ ; its mass is  $M_1$ . Fill it partly with the diluent water, then add to it the contents of the ampoule.
- d) Top up to volume  $V_2$  with the diluent water and weigh: its mass is  $M_2$ .
- e) Weigh the ampoule with its residue: its mass is  $m_1$ .
- f) Calculate the exact value of the dilution factor by the relation:

$$D'_i = \frac{M_2 - M_1}{m_2 - m_1}$$

This process should be repeated for each dilution operation.

It is necessary to use a highly accurate balance for the weighing operations. Such equipment is costly, but the gravimetric method can give very accurate results for large dilution factors. This method is particularly recommended for radioactive tracers, because it permits remote handling and with these tracers the number of standard dilutions is much reduced.

### 6.4 Influence of background — Clarification

#### 6.4.1 Background noise

If the conditions of preparation of the standard dilutions  $D'_i$  are such that they are strictly comparable with the samples taken from the sampling cross-section, there is no need to take account, in calculation of the discharge, of the initial concentration  $c_0$  of tracer, or of a substance having similar analytical characteristics, existing in the flow before the addition of  $c_2$ , as long as it can be considered constant over the duration of the measurement. It is sufficient if the standard dilutions are prepared with water taken from the flow to be measured, either upstream of the point of injection during the measurement, or in the sampling cross-section before the measurement. The procedure recommended in 6.3 permits the calibration of the measuring instrument, taking into account implicitly the effect of  $c_0$ .

A sample of background water with no tracer added may be used as a blank, which should be treated in exactly the same way as the other samples to be analysed; it is then possible to estimate the overall “background noise” of the analysis, which may partially be caused in particular by the presence of tracer in the flow.

This operation is essential where standard dilutions have not been made up with water taken from the flow. However, as this blank is usually equivalent to a very high value of the dilution factor, the reading corresponding to it is far outside the working range adopted for the determination of  $D$ . A significant risk of inaccuracy can thus result and this reading cannot be used in the determination of the response function  $G' = G'(c')$ . This result is nevertheless a useful element in the interpretation of the analysis.

#### 6.4.2 Sample pretreatment (clarification of samples and standard dilutions)

Where samples are brought from the field to the laboratory for analysis, certain tracers may be subject to losses caused by adsorption onto organic or inorganic sediments. Adsorption problems encountered in the use of specific tracers are detailed in ISO 9555-2, ISO 9555-3 and ISO 9555-4. However, the practice of decantation and filtration recommended applies to any tracer, and is outlined in this subclause.

Adsorption onto sediment is most likely to be a problem with finely-divided suspended material, which has a high surface area. If the river water contains suspended solid matter, several remedies may be considered.

- a) A simple decantation may be sufficient to clear the samples. In this case the standard dilutions shall be made up with background water carefully agitated beforehand. The flasks containing samples and standard dilutions shall be of identical shape, and the decantation times shall also be identical.
- b) If decantation does not suffice to clear the water, samples and standard dilutions prepared with turbid background water may be filtered, as soon as possible after collection, using a portable vacuum filtration unit or syringe-mounted filters. Glass-fibre filters capable of removing all material larger than  $2\ \mu\text{m}$  are suitable for general use; smaller particles, down to  $0,45\ \mu\text{m}$ , can be removed by membrane filters. It is necessary to change the filter if it becomes clogged, or if samples of different concentrations are to be filtered, and to saturate the filter with the solution to be filtered before each operation. For this purpose a certain quantity of the solution to be analysed is filtered and discarded, only the liquid subsequently filtered being retained for measurement.
- c) If the water is laden with colloidal clay material and clogs the filters rapidly, it is possible to accelerate the preliminary decantation by adding to the samples a known volume of a solution of aluminium sulfate for simple cases, or of sodium silicate followed by aluminium sulfate for the more difficult cases.

Preliminary tests will be necessary with all sample treatments to establish that no apparent or real changes in tracer concentration result, for example from uptake or release of tracer by the filter media. It is recommended that samples and standard solutions be treated in a rigorously identical fashion.

#### 6.5 Accuracy and limitations of the dilution method

The general formula for the calculation of discharge

$$Q = kD$$

can be written

$$Q = kD'r_c$$

where

$D'$  is the standard dilution closest to the experimental value  $D$ ;

$r_c$  is a correction coefficient (often close to unity) determined by the measuring instrument.

It is clear that the uncertainty in  $Q$  is the combination of the uncertainties in  $k$ ,  $D'$  and  $r_c$ .

The uncertainty in  $k$  ( $q$  or  $V/T_p$ ) is generally easily reduced by using classical methods of repeated sampling and measurement (see 9.3 and 10.2).

The uncertainty in  $D'$  is one of the factors limiting the field of application of the method. In fact, for a given tracer, injection device and measuring instrument the maximum measurable discharge is proportional to the ratio.

$$D' = \frac{c_{1\ \text{max}}}{c_{2\ \text{min}}}$$

where

$c_{1 \text{ max}}$  is the maximum injectable concentration;

$c_{2 \text{ min}}$  is the minimum measurable concentration.

This ratio can reach values in excess of  $10^7$  with certain tracers. It is necessary therefore to prepare standard dilutions of this order with an accuracy approaching 1 % (gravimetric method or volumetric method followed meticulously, see 6.3.4).

The uncertainty in  $r_c$  depends on the method of analysis, on the instrument and on the concentration  $c_2$ , and it increases as  $c_2$  approaches  $c_{2 \text{ min}}$ . It also involves the sensitivity of the analysis, which can be defined as the ratio,  $\Delta G/\Delta c_2$ , of the change in response of the instrument to the change in concentration around the usual values of  $G$  and  $c_2$  analysed.

## 7 Choice of tracer

The tracers to be used in practice shall conform as closely as possible to the ideal tracer, whose characteristics are described in 7.1.

### 7.1 Characteristics required of a measuring tracer

#### 7.1.1 Hydraulic and physicochemical characteristics

A good flow tracer shall have a hydrodynamic behaviour identical with that of molecules of water. This property shall not be altered by a change of chemical form or possibly of phase due to an interaction with the medium, nor by variations of temperature which can occur in the course of the gauging.

When the application of a tracer requires the use of a solvent, the solution obtained shall have a density close to that of the flow, to avoid segregation in the zone around the injection point.

The tracer shall not be retained by parts of the channel (material or vegetation) nor by material in suspension in the flow, in such proportions as to affect the desired accuracy of the measurement.

The tracer shall be stable in very dilute solution in natural waters under the influence of light and temperature, so as to permit a delay in analysis which may arise from the remoteness of certain gauging sites.

The tracer shall have the greatest possible solubility in water, to ensure convenience of use in gauging.

The tracer shall be analysable at the lowest possible concentration, to extend its use to the measurement of large flows.

#### 7.1.2 Metrological characteristics

There shall be a well-defined physical or chemical relationship  $G = G(c)$  between a measurable quantity  $G$  and the concentration  $c$  of the tracer in aqueous solution.

This relation shall be stable and precise over a large range of concentration around the usual value  $c_2$  for samples taken from the sampling cross-section.

Examples are: the disintegration rate of a radioactive tracer, the Beer-Lambert Law for tracers analysed by colorimetry, the relation between concentration and conductivity for certain chemical tracers, the intensity of fluorescence for certain dye tracers, etc.

It shall be possible to establish this relation with an instrument which shall combine the following essential qualities:

- high sensitivity to low values of  $c_2$ ,
- high stability over the period of analysis.

It is also desirable that the apparatus be sturdy, proven, easily available and of reasonable cost, and that it can be operated by a technician who is trained but not specialized, and maintained easily and inexpensively.

## 7.2 Criteria for comparison

### 7.2.1 Preliminary note

The choice of tracer depends on whether the aim is to find a tracer and an associated analytical procedure which covers a vast field of application and can be used for a large number of measurements, or on the contrary to find the optimal solution for a particular isolated application.

In these two cases the tracer shall satisfy the criteria enumerated in 7.1, but it is necessary, in making a choice, always to associate three elements: tracer, method of analysis and analytical instrument.

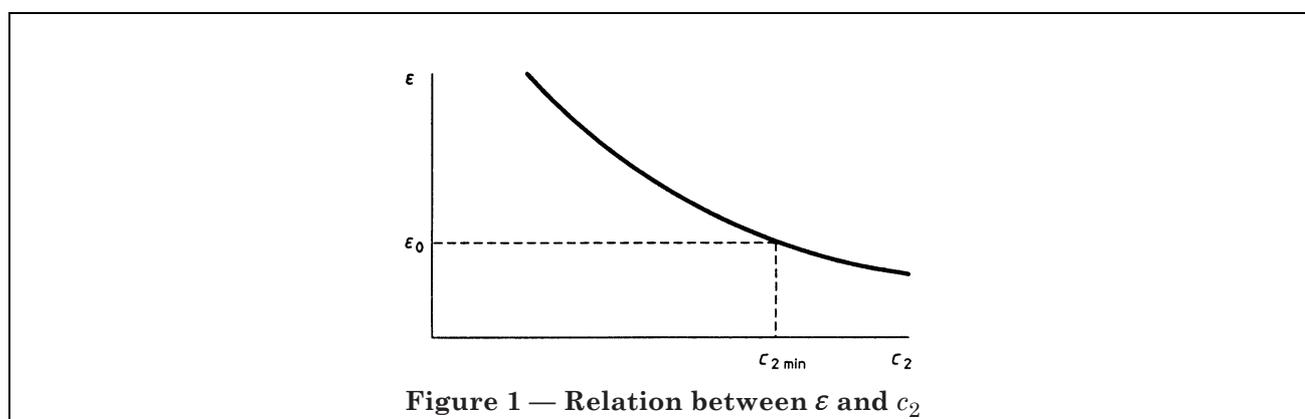
**7.2.2 Definition of criteria**

**7.2.2.1 Minimum measurable concentration,  $c_{2 \text{ min}}$ , at a level of accuracy  $\epsilon$  fixed at a given probability  $P$  (for example 95 %)**

For each combination of tracer, method of analysis and analytical instrument, the relation  $\epsilon = f(c_2)$  is established, in which  $\epsilon$  is defined by the expression (see also Figure 1):

$$\epsilon = \frac{(T_1, T_2)_P}{c_2}$$

where  $(T_1, T_2)_P$ , is the confidence interval associated with a given probability  $P$ .



**Figure 1 — Relation between  $\epsilon$  and  $c_2$**

This relation, which involves the total number  $n$  of samples adopted for the measurement, permits the determination of the concentration  $c_{2 \text{ min}}$  associated with a given value  $\epsilon_0$  (for example 1 %). In the comparison of different tracers, account is taken of the values of  $c_{2 \text{ min}}$  so defined for fixed values of  $n$ ,  $P$  and  $\epsilon_0$ .

**7.2.2.2 Maximum dilution factor**

It is possible to calculate, for each combination of tracer, method of analysis and analytical instrument, the ratio

$$D_{\text{max}} = \frac{c_{1 \text{ max}, \theta}}{c_{2 \text{ min}, \epsilon}}$$

where

$c_{1 \text{ max}, \theta}$  is the maximum concentration in water at  $\theta$  °C ( $\theta$  is the lowest temperature that is likely to be experienced by the injection solution);

$c_{2 \text{ min}, \epsilon}$  is the minimum measurable concentration at  $\epsilon$  %.

This relationship characterises the suitability of the given procedure for the measurement of large flows, and the convenience of its use.

**8 Choice of measuring reach**

**8.1 General considerations on the choice of a site**

The measuring reach shall be of length at least equal to the mixing length corresponding to the discharge to be measured (see 8.1.1). There shall be no disturbing gains or losses of tracer or of water in the measuring reach.

It is also desirable that the distance between the injection cross-section and the sampling cross-section be as small as possible: this leads to savings of time and tracer. In consequence, it is necessary to choose a reach where the river is as narrow and as turbulent as possible, where there are no dead water zones, and with numerous transverse currents to promote good lateral mixing, which is the most difficult to achieve. Vegetated zones, and zones where the river separates into branches, shall be avoided.

In all cases it is preferable to inject into a zone of high velocity flow, if possible towards the opposite bank if the injection is performed at one bank.

The sampling cross-section should be chosen in a narrow zone where there are no return currents and no dead water zones.

### 8.1.1 Mixing condition

The fundamental theoretical condition for the application of the dilution method is that the tracer should be adequately mixed with the river water at the sampling cross-section.

For the method of sudden injection, this condition can be written:

$$\int_{t_0}^{\infty} c_2(t) dt = \text{constant at all points of the section}$$

For the constant rate injection method, this identical condition is reduced to:  $c_2$  is constant over the cross-section at the steady state, plateau condition.

In practice, the impossibility of obtaining perfect mixing in the field leads to the introduction of the concepts of degree of mixing (see 3.12) and mixing length (see 3.13).

The degree of mixing  $x$  cannot be measured exactly by multiple sampling in the measuring cross-section without preliminary knowledge of the distribution of flow in this cross-section. Nevertheless, for a degree of mixing higher than 90 % and  $m$  equally-spaced sampling points across the river, the following formulae, giving each sample equal weight, can be used as an approximation:

a) In the case of the sudden injection method, the degree of mixing is

$$x = 100 \left( 1 - \frac{\sum_{i=1}^m |A_i - \bar{A}|}{2m\bar{A}} \right)$$

where

$$A_i = \int_{t_0}^{t_0 + T_p} c_{2i}(t) dt$$

$$\bar{A} = \frac{1}{m} \sum_{i=1}^m A_i$$

b) In the case of the constant rate injection method, the degree of mixing is

$$x = 100 \left( 1 - \frac{\sum_{i=1}^m |c_{2i} - \bar{c}_2|}{2m\bar{c}_2} \right)$$

To obtain the greatest accuracy in the flow measurement, it is necessary to have the largest possible value of  $x$  in the sampling cross-section.

For  $x$  greater than 90 %, the expected error in the flow can be as great as  $2(100 - x)$  %. For this reason, a degree of mixing of at least 98 % is recommended for most measurements. It is also important to note that, to improve the mixing from  $x = 90$  % to  $x = 95$  %, the length of the measuring reach must be multiplied by about 1.4, and that a nearly identical multiplying factor is necessary to increase  $x$  from 95 % to 98 %.

## 8.1.2 *Special cases*

### 8.1.2.1 *Dead water zones*

If dead water zones are present, they can retain tracer and release it only very slowly. In the case of the constant rate injection method, the time for the establishment of the plateau concentration is thereby prolonged, and as a consequence so is the time taken for the gauging. In the case of the sudden injection method the time of the gauging is also prolonged, as it is necessary to continue measurements until all the tracer retained in the dead water zones has passed through the sampling cross-section. This can cause an error in the measurement, because a non-negligible quantity of tracer can pass the downstream section at the end of the test, when concentrations are too low to be measured.

### 8.1.2.2 *Inflow of water into the measuring reach*

Measurements can be made when there is an inflow of water (tributaries or springs) in the measuring reach only if good mixing is achieved in the sampling cross-section. The measured discharge then includes the intermediate inflows of water.

### 8.1.2.3 *Diversion or leakage from the measuring reach*

If there is a leakage or diversion between the injection cross-section and the sampling cross-section, the result will be questionable, except when the location of this leakage or diversion is perfectly known and is situated at a point where good mixing has already been achieved. In this case, analysis of samples will lead to a value of the discharge of the river upstream of this leakage or diversion and not to the value of the discharge of the channel in the sampling cross-section.

## 8.2 *Preliminary tests and calculations*

These are required to estimate the mixing length, the corresponding duration of injection or sampling, as the case may be, and also the amount of tracer to be injected.

### 8.2.1 *Estimation of the length of the measuring reach*

#### 8.2.1.1 *Experimental determination*

A first trial can be carried out using a strongly coloured dye. A concentrated solution of this dye can be injected over a relatively short time at a point situated at the beginning of the measuring reach. Visual examination of the dispersion of the solution makes it possible to determine whether there are any dead water zones or other zones of tracer loss, and is a first indication of the shortest distance which shall separate the point of injection and a convenient sampling cross-section.

This procedure can be improved by continuous injection using a fluorescent tracer, whose transverse distribution of concentration is studied at different sections downstream, including flow close to the banks, with the aid of a portable fluorimeter. Detailed procedures for the use of fluorescent tracers are the subject of ISO 9555-4.

#### 8.2.1.2 *Estimation of mixing length*

##### 8.2.1.2.1 *Alluvial streams*

For alluvial streams or canals, the distance  $L$ , in metres, required for 98 % mixing can be estimated using the following equation:

$$L = \frac{k\bar{v}\bar{b}^2}{E}$$

where

- $k$  is a coefficient equal to 0,1 for a central injection, or 0,4 for a side injection (the values of  $k$  for various types of injection and degrees of mixing  $x$  are presented in Table 1);
- $\bar{v}$  is the mean velocity in the reach, in metres per second;
- $\bar{b}$  is the average stream surface width between the injection and sampling cross-section, in metres;
- $E$  is the transverse mixing coefficient, in square metres per second.

$E$  is given by the formula:

$$E = 0,2\bar{d}v^*$$

where

$\bar{d}$  is the average stream depth between the injection and sampling cross-sections, in metres;

$v^*$  is the shear velocity, in metres per second, given by

$$v^* = (gr_h S)^{1/2}$$

or by substituting  $r_h$  with  $\bar{d}$ ,

$$v^* = (g\bar{d}S)^{1/2}$$

where  $g$  is the acceleration due to gravity (9,81 m s<sup>-2</sup>).

Thus one obtains:

$$L = \frac{k\bar{v}\bar{b}^2}{0,63S^{1/2}\bar{d}^{3/2}}$$

**Table 1 — Values of the coefficient  $k$**

Type of injection	Values of $k$ , for a degree of mixing, $x$ , of			
	80 %	90 %	95 %	98 %
Central injection	0,032	0,050	0,070	0,10
Side injection	0,13	0,20	0,28	0,40
Multiple injection				
2 points	0,007 5	0,012	0,017	0,025
3 points	0,004 1	0,006 3	0,004 5	0,011

It is assumed that in a 2 or 3 point multiple injection, each injection is made at the centre of the half or third of the flow associated with the injection point.

**8.2.1.2.2 Mountain streams**

Often, the mixing distance in mountain streams or streams with many contractions and/or bends will be shorter than computed by the equation given in 8.2.1.2.1. The mixing length for these types of channels may best be estimated by an empirical relation developed for the area in which the measurements are to be made. One such relation that has been used with success is as follows:

$$L = a\bar{b}Q^{1/3}$$

where

- $L$  is the mixing distance, in metres;
- $Q$  is the discharge, in cubic metres per second;
- $a$  is a coefficient, varying from 8 to 28 (a value of 10 is used for most conditions, but a higher value should be used for very steep turbulent streams);
- $\bar{b}$  is the average stream surface width between the injection and sampling cross-sections, in metres.

NOTE 3 Any formula can only provide an estimate of the distance required for mixing. *In situ* tests are required for a more exact determination of  $L$ .

### 8.2.2 Duration of injection and sampling

#### 8.2.2.1 Constant rate injection method

The duration of injection shall be such that a steady-state concentration is established in the sampling cross-section for a sufficient time,  $T_c$ , to allow a sampling scheme corresponding to the conditions set out in 9.4.

The duration of injection,  $T_i$ , will depend in general on turbulence in the measuring reach, varying directly with the length of the reach and the extent of dead water zones, and inversely with the mean velocity of the water. In practice,  $T_i$  is determined from the preliminary trial (defined in 8.2.1.1) for the determination of mixing length. The detection at a number of potential sampling cross-sections, using a fluorimeter, of an injected fluorescent tracer, or more simply the visual observation by trained observers of the time of passage of a colour cloud, and the multiplication of this period by a certain coefficient based on experience, can be used to estimate  $T_p$ , the time of passage of tracer through these cross-sections. It is then possible to construct curves (1 and 2 in Figure 2) corresponding to the appearance and disappearance of tracer in each section under consideration in response to a sudden injection.

Let the sampling cross-section, corresponding to the minimum mixing length compatible with the desired accuracy, be  $S_1$ .

To determine the duration of injection required to ensure a plateau of concentration of duration  $T_c$  at the selected sampling cross-section, it is necessary to add the required plateau duration to the time of passage  $T_p$  through  $S_1$ .

The variation of the plateau duration for stations downstream can be seen by drawing a curve 1', parallel with curve 1 of appearance of tracer, and offset by the sum of the duration of plateau  $T_c$  and the time of passage through  $S_1$ ,  $T_p$  (see Figure 3). The intercept of this curve with the time-axis is the minimum duration of injection. Curve 2' parallel to curve 2 gives the time of cessation of passage of the tracer at each cross-section. It is easily seen from the figure that the intersection of curves 1' and 2 marks a cross-section downstream of which there is no plateau of concentration at all.

#### 8.2.2.2 Sudden injection method

In the same way as for the constant rate injection method, the preliminary test is used to determine the start and duration of sampling in the selected cross-section. As a precaution, this duration shall be chosen larger than the duration determined by visual inspection of colour. It may be necessary to adjust the duration of injection to obtain a satisfactory duration of sampling, taking into account the following factors.

For a given mass of tracer injected, the mean concentration noted at a point in the sampling cross-section during the time of passage decreases as the duration of the injection increases. As this value of mean concentration affects the accuracy of the measurement, it may be preferable to perform a short injection so as to reduce the mass of tracer required. However, it may be necessary to increase the duration of injection, and consequently the time of passage of the tracer cloud through the sampling cross-section, to ensure sufficient accuracy in certain modes of sampling (see 10.4.2).

In natural streams it is not possible to calculate  $T_p$ , except in a very approximate fashion. However, experience shows that the time of passage  $T_p$  of the tracer is larger than the square root of the time  $t_a$  between the injection of tracer and its arrival at the sampling point, these times being expressed in hours.

### 8.2.3 Determination of the mass of tracer to be injected

The mass of tracer to be injected, and hence the corresponding volume of injection solution, are determined on the one hand from the results of preliminary trials (see 8.2.1.1), and on the other hand as a function of the characteristics of the injection and sampling equipment, factors pertaining to the tracer and to the selected method of analysis and the estimate of the discharge to be measured.

#### 8.2.3.1 Estimation of the discharge to be measured

An estimate  $Q'$ , however rough, of the discharge  $Q$  to be measured is necessary, as much for field operations as for analysis by the standard dilution method.

For this purpose it is necessary to use either a direct estimation of the mean velocity (by floats or natural floating bodies) and the wetted section, or indications from a neighbouring hydraulic structure (sluices, hydroelectric stations, etc.) or, obviously, from the stage-rating curve of the nearest gauging station on the same stream, or the results of a preliminary test to estimate the mean velocity. It is sufficient to obtain an approximate value such that  $Q'/2 < Q < 2Q'$  so as to avoid the most difficulties.

#### 8.2.3.2 Constant rate injection method

The tests lead to an estimate,  $Q'$ , of the discharge to be measured, and the determination of  $T_i$ , the minimum duration of injection: it is also necessary to choose an approximate value  $c_2$  of the resulting sample concentrations, which corresponds to the range of optimum accuracy and sensitivity of the analytical procedure.

The mass of tracer to be injected is given by the relation:

$$M = Q' T_i c_2$$

The injection rate and the concentration of the injection solution are deduced from the relation (2):

$$q c_1 = Q' c_2 = M/T_i$$

NOTE 4 If  $q_{\min}$  and  $q_{\max}$  are the limits of the range of injection rates available for the equipment in use for the gauging to be possible it shall be feasible to choose  $q$  such that

$$q_{\min} < q < q_{\max}$$

and

$$\frac{Q' c_{2\min}}{c_{1\max}} < q < \frac{V_{\max}}{T_i}$$

where

$V_{\max}$  is the maximum volume of solution that can be injected conveniently at constant rate under the conditions of measurement;

$c_{1\max}$  is the maximum tracer concentration practically realizable under the conditions of measurement;

$c_{2\min}$  is the minimum concentration that can be determined precisely by the analytical equipment.

If this double condition is satisfied, it is possible to choose a pair of values  $q$  and  $c_1$ , taking into account the following additional criteria: it is of advantage to minimize the ratio  $c_1/c_2$ , the dilution factor, since the accuracy of preparation of the standard dilutions varies inversely with this factor. Hence for a fixed  $c_2$ , the minimum value of  $c_1$ , which corresponds to the largest possible injection rate, is found:

$$q < V_{\max}/T_i$$

$$c_1 = Q' c_2/q$$

The measurement is impossible if

$$\frac{Q' c_{2\min}}{c_{1\max}} > \frac{V_{\max}}{T_i}$$

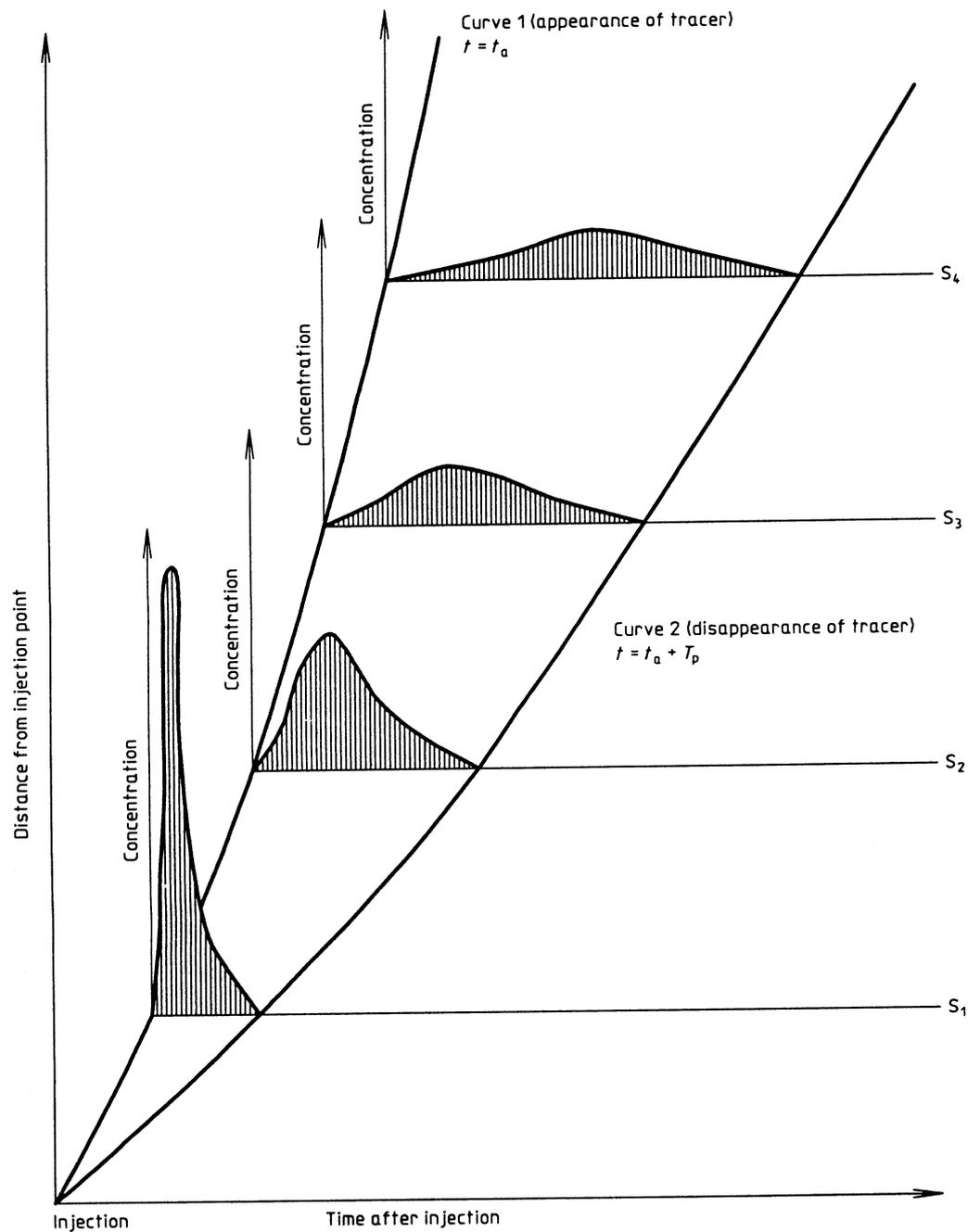
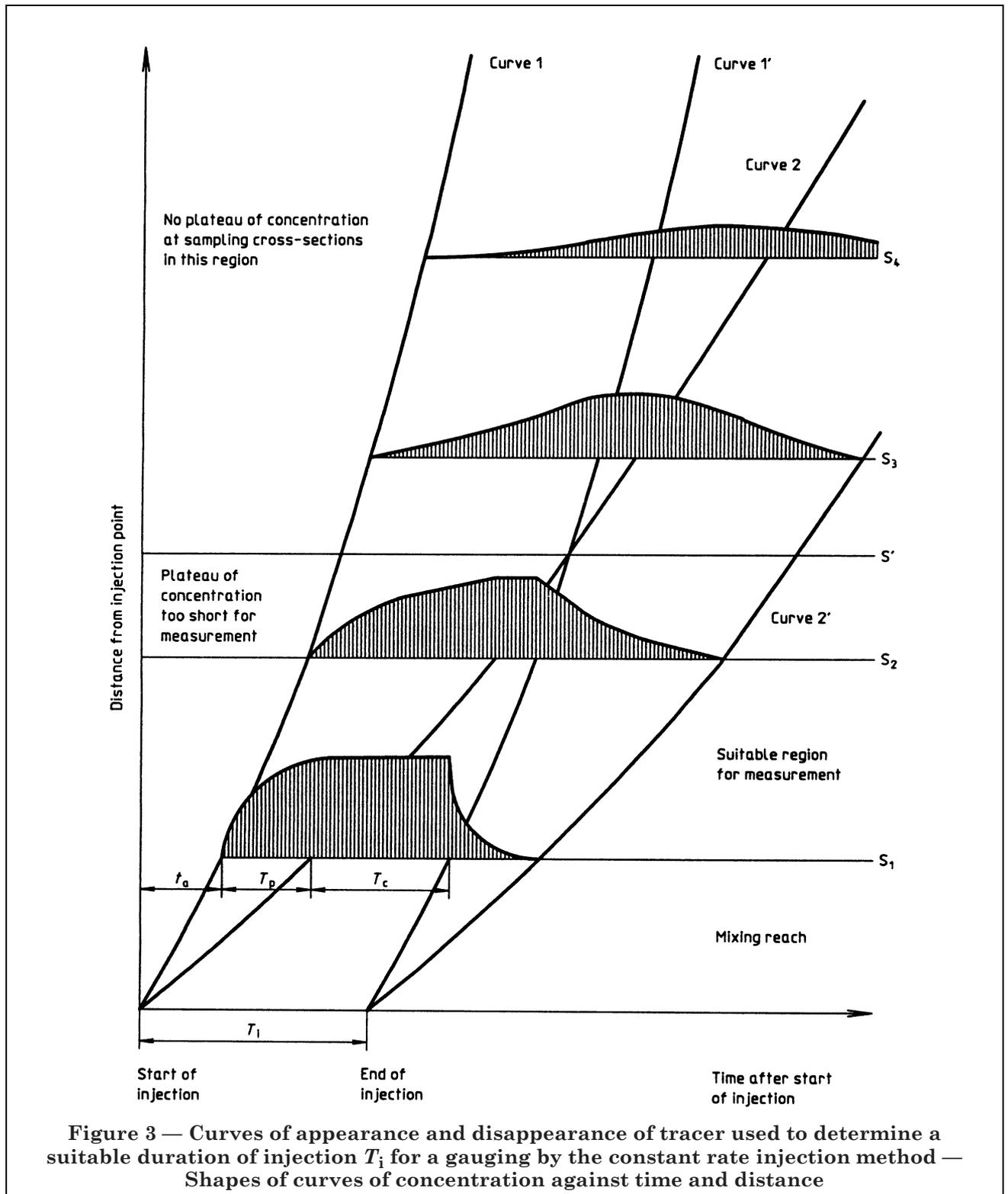


Figure 2 — Curves of appearance and disappearance of tracer for a sudden injection — Shapes of curves of concentration against time and distance



In this case, it may be possible to make a measurement by the sudden injection method (see 8.2.3.3).

NOTE 5 The mass of tracer to be injected is a function of  $T_i$ , other factors held constant. It is clearly of advantage to minimize  $T_i$ ; this imposes the condition of seeking, in the measuring reach, the best combination of injection cross-section and sampling cross-section and hence the reach offering the shortest possible mixing length. If necessary, it is possible to alter the position of the point of injection in the cross-section, to inject at many points, or on the inner bank of a bend or in a zone of strong turbulence etc.

NOTE 6 The highest possible tracer injection rate for a given set of equipment, and hence the maximum measurable discharge is given by the combination of  $c_{1\max}$  and  $q_{\max}$ ; it is clear that under these conditions the duration of injection is as short as possible (hence the advantage in following the recommendations above) but the dilution factor is maximized.

### 8.2.3.3 Sudden injection method

The preliminary tests provide a value  $Q'$  of the discharge to be measured, and a value  $T_p$  for the maximum practical duration of the passage of the tracer cloud through the sampling cross-section. A value  $\bar{c}_2$ , an optimal mean resultant concentration, is fixed corresponding to the optimum range of the analytical equipment. Applying formula (5):

$$Q = kD$$

with  $k = VT_p$  and  $D = c_1/\bar{c}_2$  results in:

$$M = c_1V = Q'T_p\bar{c}_2$$

In practice, a value larger than  $M$  is adopted for safety, to take into account the uncertainty in this computation.

### 8.2.3.4 General notes

8.2.3.4.1 In the sudden injection method, to reduce the ratio  $D$  it is necessary to inject the volume  $V_{\max}$ , but in order to reduce the mass of tracer and the corresponding injected volume, it is advantageous to seek, in the measuring reach, local conditions for injection and sampling which minimize  $T_p$ . This optimization is the same for the sudden injection method as for the constant rate injection method (see Figure 3) since the duration of injection at constant rate  $T_i$  is equal to  $T_p$  plus the time  $T_c$  chosen for the plateau of concentration  $c_2$  in the sampling cross-section:

$$T_i = T_c + T_p$$

8.2.3.4.2 It is possible to compare the masses of tracer necessary, under the same conditions, for the measurement of the same flow by the two methods:

$$M_1 = Q'T_p\bar{c}_2 \text{ (sudden injection)}$$

$$M_2 = Q'(T_p + T_c)c_2 = M_1 + Q'T_cc_2 \text{ (constant rate injection)}$$

Thus more tracer is always needed for the constant rate injection, and it is of advantage not to increase unnecessarily the length of the plateau  $T_c$ .

In fact this duration shall be sufficient to prove the reality of a plateau (the fundamental hypothesis of the application of the constant rate method).

## 9 Procedure for the constant rate injection method

### 9.1 Preparation of the injection solution

It is essential that the injected solution be perfectly homogeneous. This can be ensured by using energetic stirring with a mechanical agitator or a circulating pump. It is recommended that the injection solution be prepared, in a vessel separate from the supply vessel, with water filtered by an appropriate method. If nevertheless mixing is carried out in the supply vessel, this shall have a capacity sufficient to prevent the need to add liquid or tracer during the injection. In the case of an injection of long duration, all necessary precautions shall be taken to prevent variation in concentration of the solution with time (for example by evaporation which can occur at ambient temperature).

### 9.2 Injection of the solution

The solution shall be injected (at the selected injection cross-section in the measuring reach) at constant rate for the time necessary to obtain a plateau of concentration of sufficient duration (see 8.2.2). Various devices can be used to carry out this injection. For all these devices it shall be possible to check the constancy of the mass rate of discharge of tracer.

One way to achieve this is to monitor the constancy of the concentration and of the discharge of the injected solution. Watertightness of circuits shall be verified; impurities in suspension which could affect the flow in the injection circuits are to be eliminated.

The devices which are most often used are:

a) Constant level vessel (see Figure 4)

A constant level vessel shall have dimensions and volume sufficient to encourage complete dispersion of the flow emerging from the inlet pipe and to avoid any direct circulation of liquid between the inlet and outlet pipes and the setting up of dead water zones in the vessel. The edge of the weir maintaining the constant level in the vessel shall be sharp, horizontal and of sufficient length that the head at the outlet orifice remains constant, despite small variations in the inlet discharge. Arrangements should be made to collect solution overflowing from the vessel during a test, to prevent its reaching the channel. It is possible to obtain a range of injection discharges with a set of orifice plates of different diameters, constructed to conform with the relevant standards.

b) Pump driven by a constant speed motor (see Figure 5).

The speed shall be very carefully held constant. This speed can be either measured directly, or deduced from the supply frequency in the case of a synchronous electric motor.

c) Other devices

1) Mariotte vessel (see Figure 6).

The constant rate injection is achieved by means of an airtight vessel provided with an orifice plate or nozzle at its bottom portion. The liquid flows through the restriction and air enters the vessel through a tube, maintaining atmospheric pressure at the lower end of the tube, set at a determined height above the restriction. The head on the restriction and consequently the discharge from the vessel remain constant, independently of the level in the vessel.

2) Floating siphon (see Figure 7).

The solution is taken from a vessel by means of a siphon that is fixed on a float. The lower end of the siphon is fitted with an orifice plate or nozzle. The head on the restriction and consequently the discharge from the vessel remain constant, independently of the level in the vessel.

### 9.3 Measurement of injection rate

The measuring device for the injection rate shall have an accuracy compatible with the overall accuracy desired for the flow measurement.

Two cases can be considered:

a) The injection device has its own measuring device whose operation conforms to the specifications of a standard (e.g. orifice plate), or which can be calibrated under conditions identical with those of the measurement.

NOTE 7 If an orifice plate is used, the flow  $q$  is influenced by the kinematic viscosity of the injection solution. This effect, generally small, shall be taken into consideration for precise measurements.

b) The injection device does not have a measuring device but the discharge may be determined during injection by the direct measurement of fundamental quantities (e.g. the fall in level in a vessel or a volumetric pump). The injection device shall not be deformed under field conditions, and can itself be calibrated outside the period of injection. It may be necessary for highly precise measurements to verify the calibration before and after the measurement.

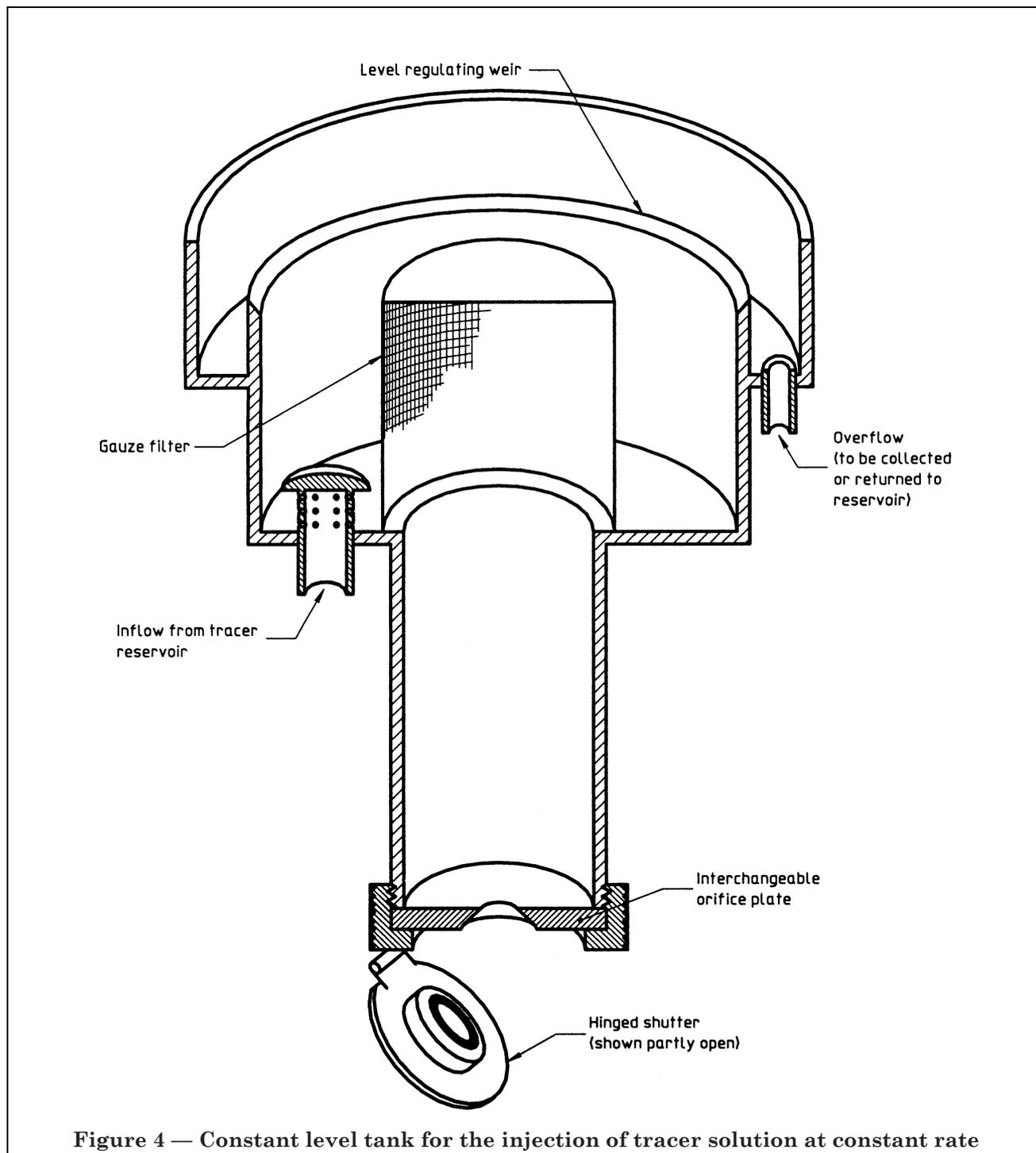


Figure 4 — Constant level tank for the injection of tracer solution at constant rate

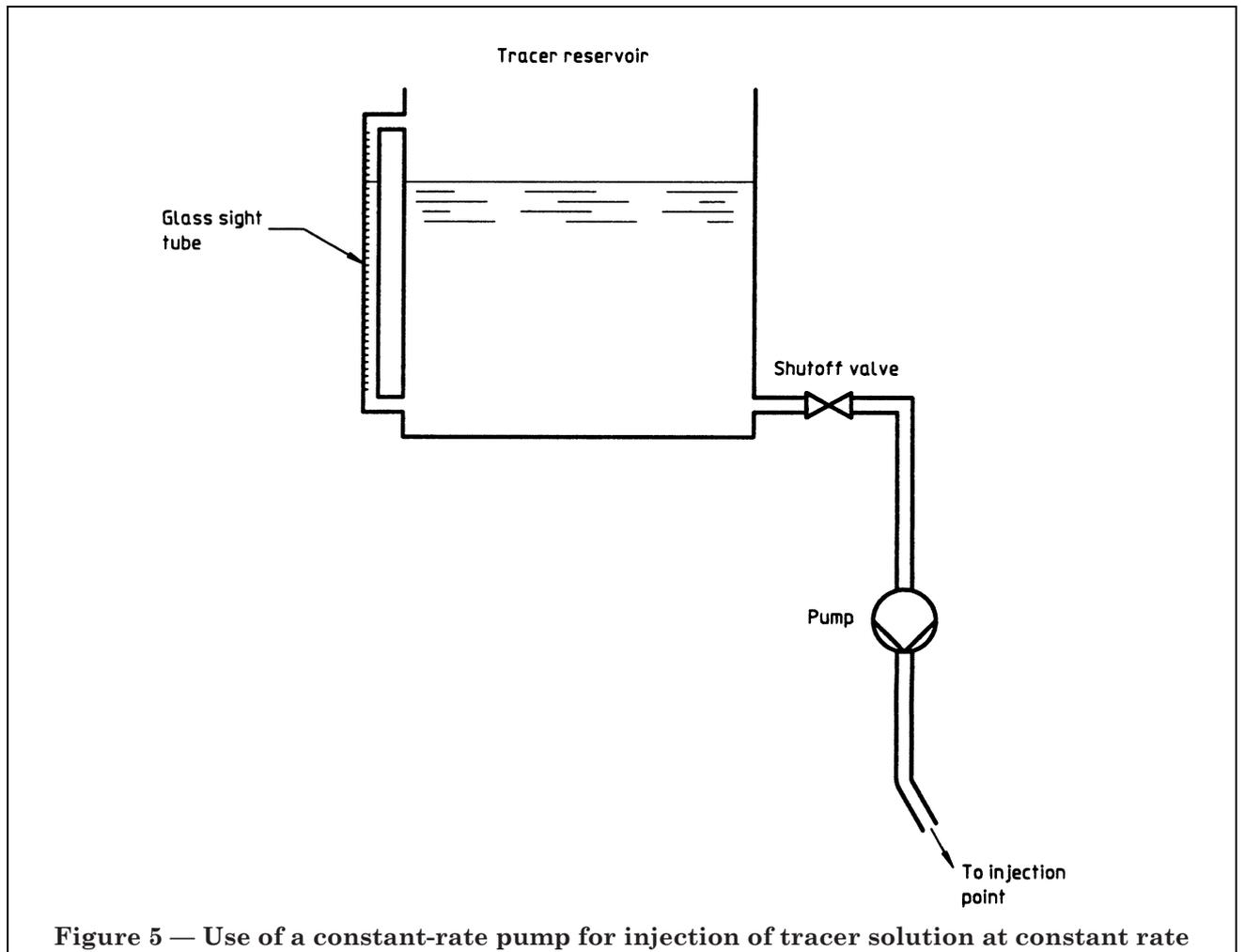
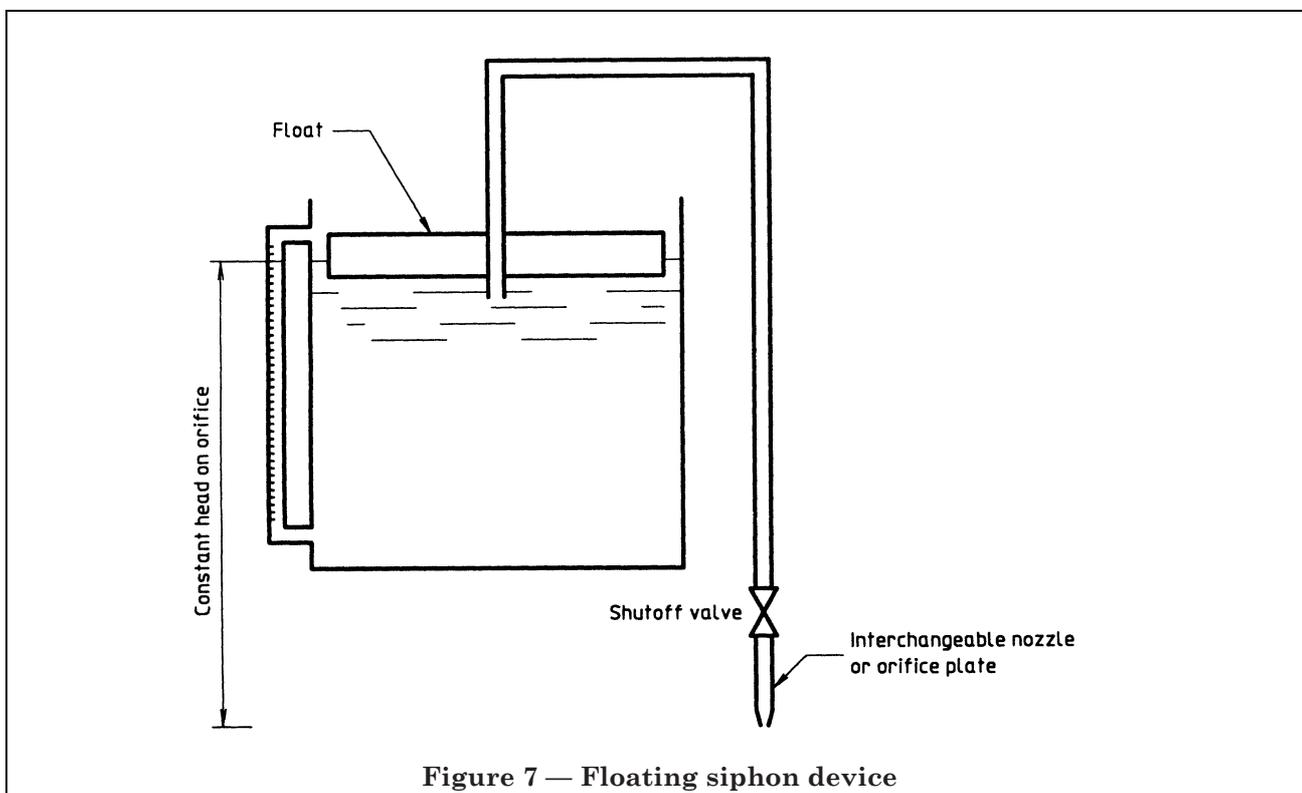
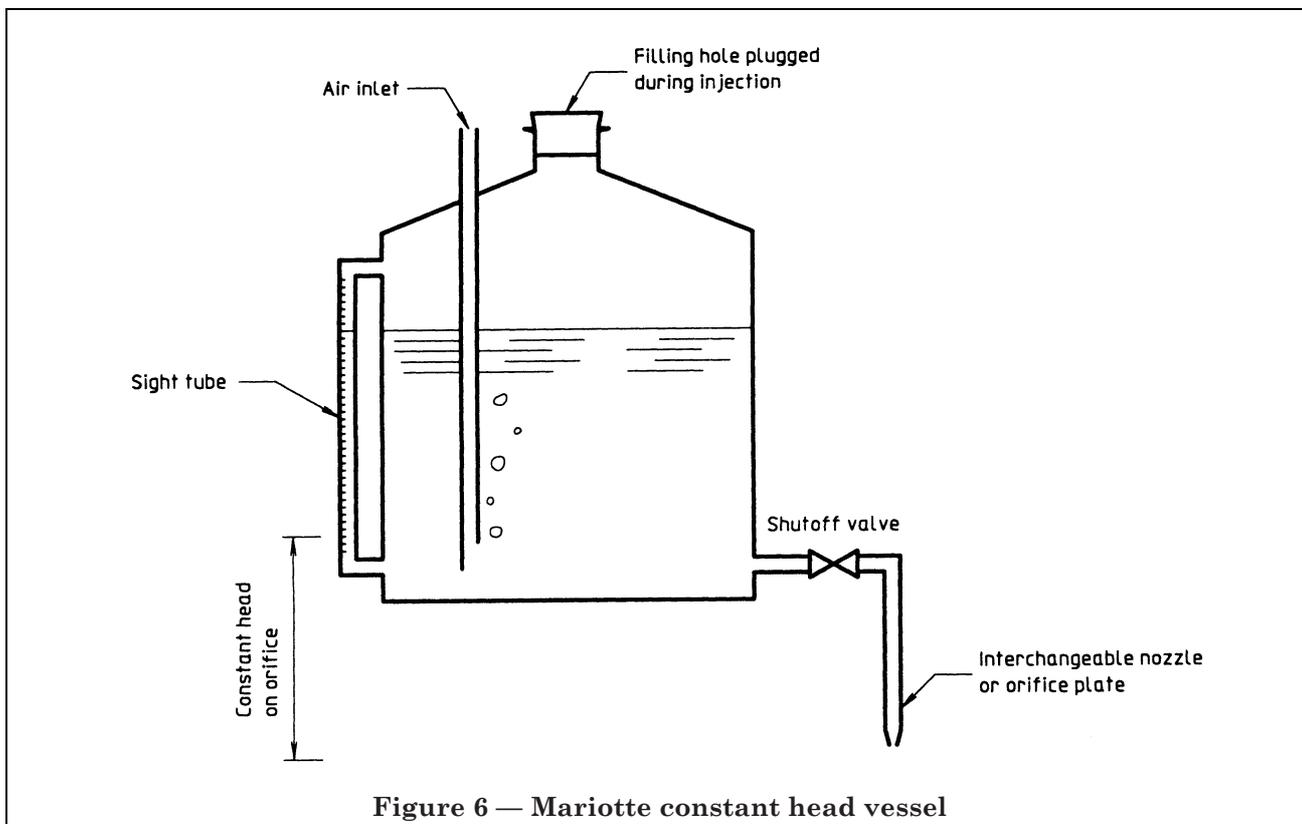


Figure 5 — Use of a constant-rate pump for injection of tracer solution at constant rate



## 9.4 Sampling

Sampling consists of the collection from the flow, either directly (by weighted bottles, for instance) or indirectly (by sampling at the outlet of a pump for example), of a certain number of samples at defined intervals of time, whose precise length, however, is not important. For the purposes of analysis and calculation of discharge, it is necessary to provide for the following samples:

- a) Two or three samples of background water, which shall be collected in the sampling cross-section before injection and upstream of the injection cross-section during and after the measurement, with the purpose of verifying that the effect of background in the channel remains constant during the measurement (see 6.4). At least one of these samples should be of sufficient volume for the preparation of the standard dilutions.
- b) Samples of water containing tracer shall be collected in the measuring cross-section at at least three points (left bank, centre and right bank) to verify that good mixing has been obtained; at each point, at least three samples shall be taken over a sufficient interval of time to confirm the plateau concentration and to permit the calculation of discharge. In practice, to mitigate the effects of accidental errors in analysis, it is preferable to increase the number of samples.
- c) Two samples of the injected solution shall be taken at the outlet of the injection apparatus, before and after the injection period, to check the homogeneity of the concentration  $c_1$  and for the determination of the dilution factor  $D$ . Furthermore, if the injection equipment permits, it is recommended that a supplementary sample be taken during the injection, without modifying the injection rate  $q$ .

NOTE 8 It is important to take appropriate precautions to avoid contamination of the sampling equipment or personnel by concentrated solution. The samples of the injection solutions should be stored and transported separately from the samples of river water.

NOTE 9 The cost of taking additional samples should be compared with the risk of taking too few samples and invalidating the gauging. Further, losses or accidental errors can occur during storage, transport and analysis; it is thus preferable to take a number of samples larger than the theoretical minimum.

## 10 Procedure for the sudden injection method

### 10.1 Preparation of the injection solution

It is recommended that a solution of concentration  $c_1$  be prepared with the necessary mass of tracer  $M$  (see 8.2.3.3), with the aim of obtaining a volume suitable for the conditions of the injection. It may be necessary to select a particular solvent other than water for this operation (see 7.1.1).

For a single measurement, it is possible to use a single vessel for:

- the mixing of the tracer solution from which samples are taken for the determination of  $c'_1$ , or the preparation of standard dilutions,
- the exact determination of the injected volume  $V$ , and
- the complete and rapid injection of all the measured volume.

For a series of comparable measurements, it is possible to separate the operations of mixing and injection by using several vessels. It is practical, for example, to carry in conveniently chosen sealed vessels, known volumes of the same injection solution.

### 10.2 Determination of the injected volume

To determine the injected volume, the three following possibilities are available:

- a) use of a calibrated reservoir;
- b) geometrical determination of the volume of the injection solution contained in the reservoir;
- c) measurement by weighing. This is the most precise method. Determination of the volume in this case requires the use of the density of the injected solution at the given temperature, which should be obtained by laboratory methods, using a sample of the injected solution.

### 10.3 Injection of the solution

The duration of the injection should be as short as possible, so as to reduce the sampling time and the mass of tracer required for a given accuracy.

However, by extending the duration of injection of the tracer, possibly to several minutes, the maximum concentrations around the point of injection are reduced, which reduces the effects of density segregation.

## 10.4 Sampling

### 10.4.1 Samples

For the purposes of the measurement, it is necessary to provide the following samples:

- a) At least three samples shall be taken in the measuring cross-section before the injection and upstream of the injection cross-section before and after the measurement, to verify that the initial concentration  $c_0$  in the channel remains constant during the measurement.
- b) One or two samples of concentration  $c_1$  of the injection solution shall be taken.
- c) In the measuring section, it is possible to proceed either by continuous recording or by discrete sampling.

In the two latter cases, sampling should be performed at at least three fixed points (left bank, centre and right bank) to verify that good mixing has been achieved. Samples near the bank shall be taken from flowing water.

This recommendation is important for a measurement on an unknown reach or on a known reach under new conditions. For routine measurements where experience can guarantee the quality of mixing, a single sampling point may be sufficient.

### 10.4.2 Methods of sampling in the measuring cross-section covering the time of passage of the tracer

#### 10.4.2.1 Continuous recording

When using the conductivity method, and where variations in temperature and natural conductivity of the water are negligible, it is acceptable to record continuously the conductivity of the water during the passage of the tracer [see Figure 8 a)].

Continuous recording is also possible when fluorescent or radioactive tracers are used.

The sensing element of the measuring apparatus may be directly immersed in the flow or placed in a diversion channel fed by sampling the flow at a reasonably constant rate at a fixed point. It is then important to provide for the fixing of the sensing element in the first case, and the inlet strainer in the second case.

#### 10.4.2.2 Collection of discrete samples

The sampling interval and the number of samples shall be chosen so as to obtain no fewer than four samples on the rising slope of the concentration-time curve, between the arrival of the tracer and the peak of tracer concentration. At least 15 samples in all shall be taken: if the results of preliminary tests are not available, or too inaccurate, more samples may be required.

Two procedures are used:

##### 10.4.2.2.1 Collection of quasi-instantaneous samples at known and recorded times

Successive samples shall be taken at each sampling point of the cross-section during the time of passage of the tracer. After analysis, the curve of variation of  $c_2$  with time (or of  $c_2/c_1$  according to the method of analysis) may be traced and planimetered. The time intervals between sampling need not be equal [see Figure 8 b)].

##### 10.4.2.2.2 Collection of quasi-instantaneous samples at equal and known intervals of time

This procedure leads to the tracing of the curve in 10.4.2.2.1, but does not require the recording of the time of each sample. It also permits the use of the procedures described in 10.4.2.3.2 and 10.4.2.4, possibly as a check at a later stage in the laboratory [see Figure 8 c)].

#### 10.4.2.3 Collection of mean sample

The two following procedures are used:

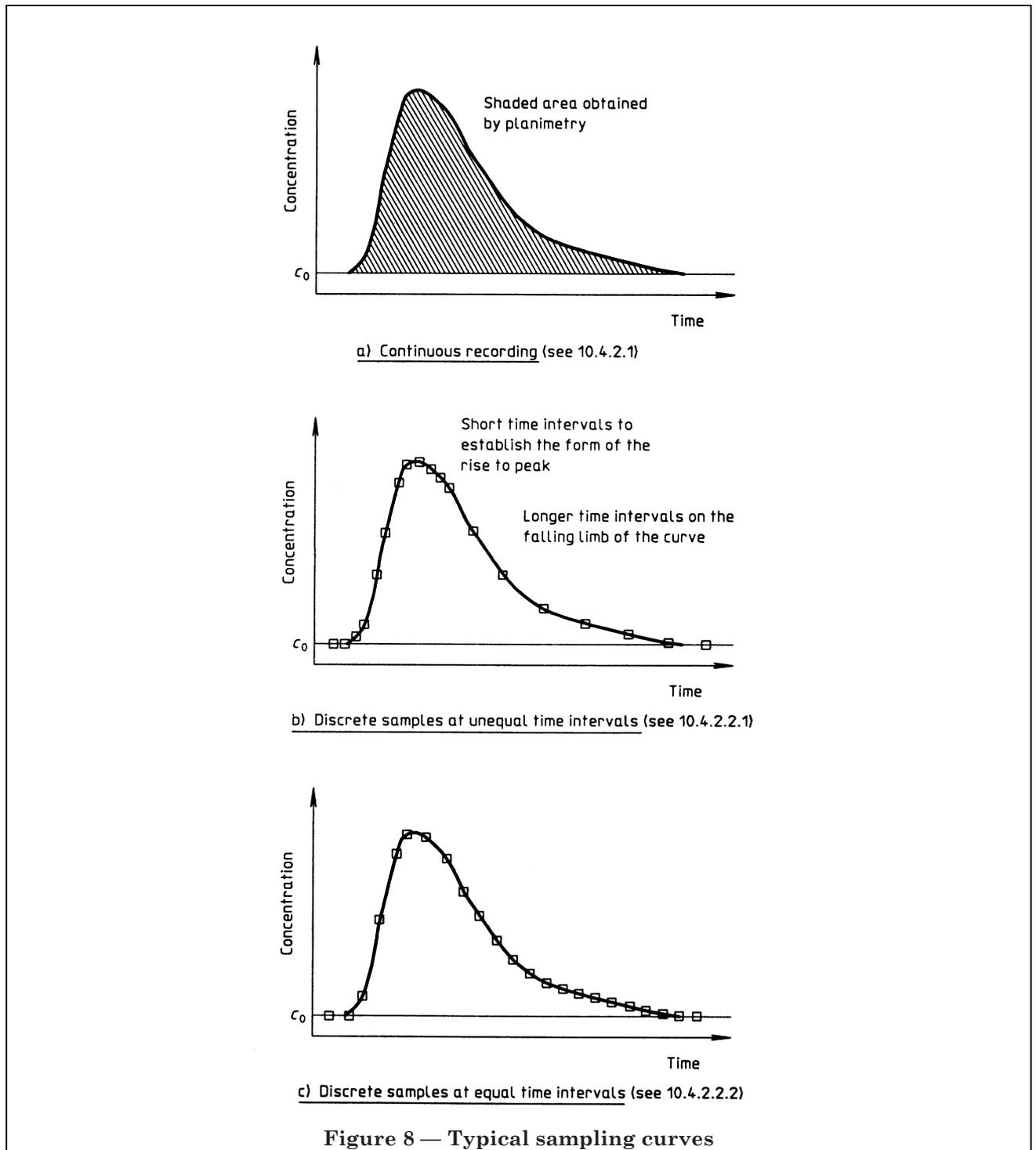
##### 10.4.2.3.1 Sampling at constant rate

Several devices can be used to obtain a sample at constant rate, during the passage of the tracer at a fixed point [see Figure 9 a)].

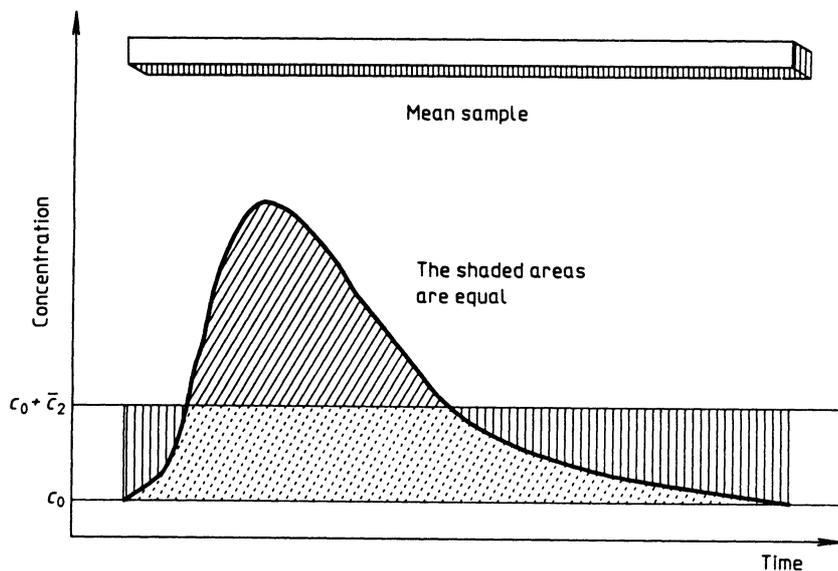
The simplest method is to siphon through a flexible pipe of inside diameter about 5 mm, provided with a strainer, into a vessel of volume sufficient to accommodate the entire sample.

If the site does not lend itself to siphoning operations, a pump may be used to supply a constant level vessel (see Figure 4). Alternatively, a volumetric pump may be used (see Figure 5).

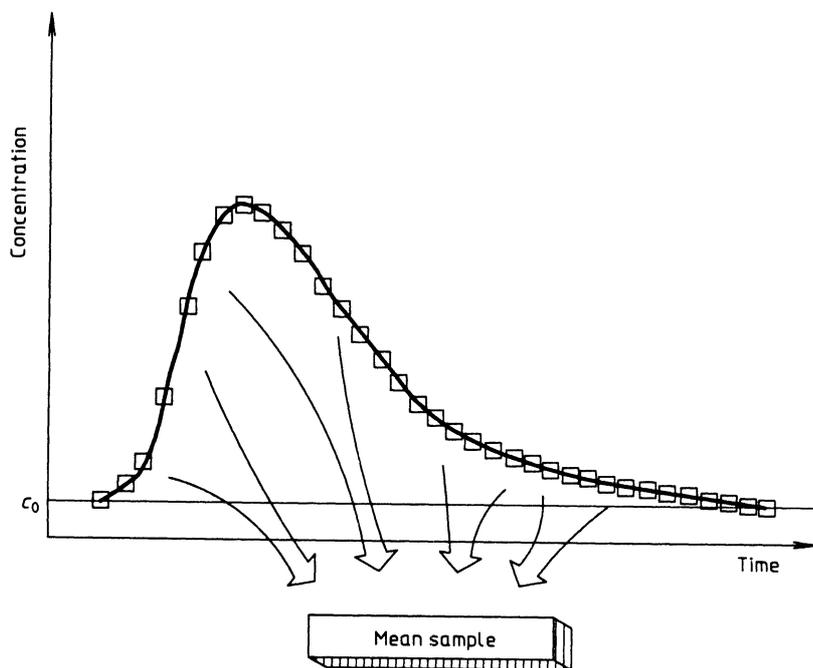
It is important, in this procedure, to fix the strainer firmly to prevent any movement.



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a) Sampling at constant rate to obtain a single mean sample (see 10.4.2.3.1)



b) Mixing of quasi-instantaneous samples taken at equal time intervals (see 10.4.2.3.2)

Figure 9 — Obtaining mean samples

**10.4.2.3.2** *Collection of quasi-instantaneous samples at equal intervals of time and mixing of these samples* [see Figure 9 b)]

To approach the conditions of **10.4.2.3.1**, the interval of time between samples should be short; the sampling interval and the number of samples shall be chosen so as to obtain no fewer than four samples on the rising slope of the concentration-time curve, between the arrival of the tracer and the peak of tracer concentration. At least 15 samples in all shall be taken: where the results of preliminary tests are not available more samples may be required.

Experience shows that departures of 5 % in the volumes of the various samples do not introduce a significant error in the calculation of the flow, if the number of samples is large.

For all the procedures, the collection of quasi-instantaneous samples at equal time intervals can be replaced by the collection of mean samples over the same time intervals. This procedure is less sensitive to variations in the rhythm of sampling and to irregularities in the rate of sampling or in the curve  $c_2 = c_2(t)$ .

**10.4.2.4** *Collection of several mean samples beginning at the same instant*

Using a pump with a manifold,  $n$  mean samples are taken at constant rate from the same fixed point in the cross-section; these  $n$  samples have the same starting time before the arrival of the tracer, and terminate at times  $t_1, t_2, \dots, t_n$  after the start of sampling [see Figure 10 a)].

The most reliable solution, which will however necessitate either the use of larger quantities of tracer or lower concentrations for the samples finishing at times  $t_1, t_2, \dots, t_n$ , is to start sampling immediately before the arrival of the tracer, and to adopt the value  $t_1 = T_p$ , where  $T_p$  is the time of passage of tracer through the reach.

In this case, if  $T_p$  has not been underestimated, all the samples will lead to the true value of discharge. If  $\bar{D}_i$  is the dilution determined by analysis for the sample which terminates at time  $t_i$ :

$$Q = \frac{V}{t_1} \bar{D}_1 = \frac{V}{t_2} \bar{D}_2 = \frac{V}{t_3} \bar{D}_3 = \dots = \frac{V}{t_n} \bar{D}_n$$

If  $T_p$  has been underestimated, the apparent discharge calculated from the first samples will be too large, but certain samples such as  $t_1 \dots t_n$ , will give the true value. In the worst case, an extrapolation of the curve  $Q(t)$  to its asymptotic value will lead to the true value of the discharge.

A more economical solution is to adopt for example:

$$t_3 = T_p \text{ with } t_1 < T_p \text{ and } t_2 < T_p \\ t_4 > t_p, t_5 > T_p \dots t_n > T_p$$

If  $T_p$  is correctly evaluated, the analysis of samples taken at times  $t_3, t_4, t_5, \dots, t_n$ , will lead to the true value of the discharge,  $t_1$  and  $t_2$  to too large a value (the sample taken at  $t_3$  will have the concentration considered as optimal for analysis) while those taken at  $t_4, t_5, \dots, t_n$ , will have lower concentrations.

If  $T_p$  has been underestimated,  $t_5$  and  $t_6$  or a prudent extrapolation of the curve  $Q(t)$  will lead to the true value of discharge.

If  $T_p$  has been overestimated, all the samples will lead to the true value of discharge.

For the application of the procedures described in **10.4.2**, it is possible to use the apparatus in Figure 11.

**10.4.2.5** *Collection of several successive mean samples*

An alternative procedure, which has the advantage of distinguishing between the systematic decline in  $Q(t)$  (see **10.4.2.4**) and random errors in the analysis, is to obtain average samples over the intervals 0 to  $t_1$ ,  $t_1$  to  $t_2$ , etc., and to analyse these samples individually [see Figure 10 b)].

If  $t_1$  is chosen equal to  $T_p$ , and if  $T_p$  is an underestimation of the true value, then the concentration of the sample taken between  $t_1$  and  $t_2$  will be greater than zero, but later samples will have concentrations tending to zero.

If the sample taken between  $t_n$  and  $t_{n+1}$  is the last that is significantly different from zero, and the samples represent dilutions  $D_1, D_2, \dots, D_{n+1}$ , then:

$$Q = \frac{V}{t_1/D_1 + (t_2 - t_1)/D_2 + \dots + (t_{n+1} - t_n)/D_{n+1}}$$

The advantage of this method is that the time  $t_{n+1}$  at which the passage of the tracer is complete is more easily and reliably estimated.

The disadvantage is that much reliance is placed on the first samples.

#### 10.4.2.6 Important notes

Whichever method is chosen, it is essential that the sampling point remains the same for all the samples and throughout the duration of sampling.

It may be desirable, in particular in the case of the method of 10.4.2.3, to have a detector placed *in situ* upstream of the measuring cross-section to indicate the time at which the first sample is to be taken. The distance at which the detector is placed shall be sufficient for sampling to start before the appearance of the tracer at the sampling cross-section. The methods of sampling in 10.4.2.3 are particularly recommended, as they involve the least cumbersome analysis. On the other hand the methods described in 10.4.2.2 can involve the use of lighter equipment in the field, and offer the possibility of analysing the effectiveness of discrete sampling methods.

In all cases, the duration of the sampling operation shall be at least equal to and synchronous with the duration of passage of the tracer cloud through the measuring cross-section (see clause 5).

## 11 Comparison between the two dilution methods

### 11.1 General

The differences between the two methods mainly concern the necessary equipment and field operations.

The constant rate injection method can provide more information on the quality of mixing and the possible effects of a variation in discharge; it is generally possible to obtain as many samples, on the plateau of concentration and distributed in time and space, as are necessary for verification. For a fixed duration of injection  $T_i$ , Figure 3 shows that the duration  $T_c$  of the plateau will be insufficient for cross-sections close to the cross-section  $S'$  corresponding to the intersection of curves 1' and 2, and will be zero for all cross-sections downstream of  $S'$ . The possible reach is therefore limited for the constant rate injection method: the method of sudden injection, in the absence of loss of tracer by adsorption, is less constrained. For this reason, and for large streams, the sudden injection method will often be preferred.

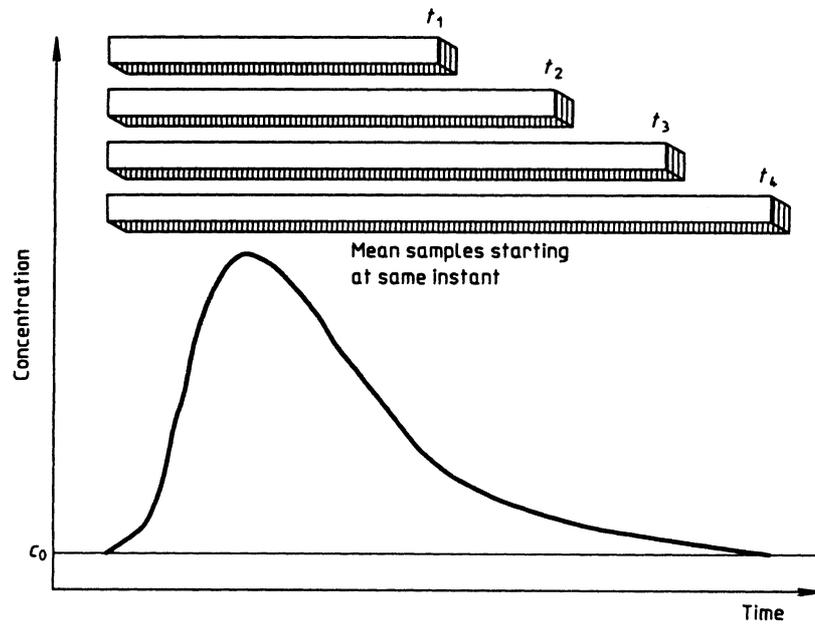
An advantage of the sudden injection method is the use of a smaller quantity of tracer, and this can be very important in terms of cost or public acceptability, in particular when diversion of water downstream of the injection point requires a certain stream water quality. However, it should be noted that peak values of concentration of tracer in the river will be higher if, for analytical reasons, the mean value for the sudden injection method is chosen to be of similar magnitude to the plateau value for the constant rate method.

The total duration of a sudden injection measurement will always be shorter than that of the same measurement by the constant rate injection method; however, the difference will only be significant for small streams.

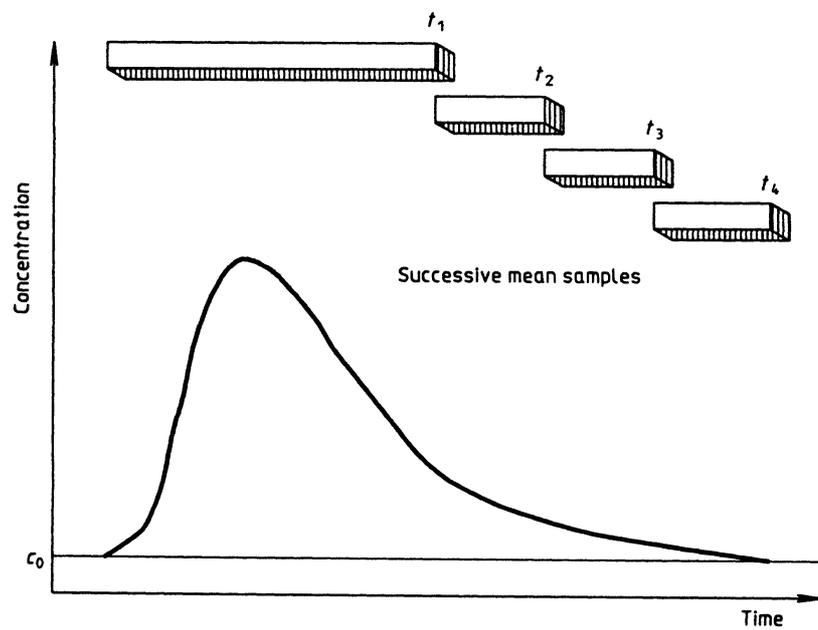
### 11.2 Injection of tracer

The constant rate method requires more sophisticated equipment for injection of tracer, to obtain a constant discharge which can be determined precisely, but conversely there is no need for precise measurement of times or volumes in the field.

The sudden injection method, on the other hand, requires no complex injection equipment, and this is an important advantage, in particular when highly active radiotracer solutions are to be injected. Nevertheless, the volume injected must be known precisely, and moreover the higher concentrations of tracer encountered near the injection point may cause problems of toxicity and density segregation; these can be diminished by increasing the duration of the injection. In this case, the rate of injection of tracer for a non-instantaneous injection need not be constant, provided that precise measurement can be made of the volume of solution injected.

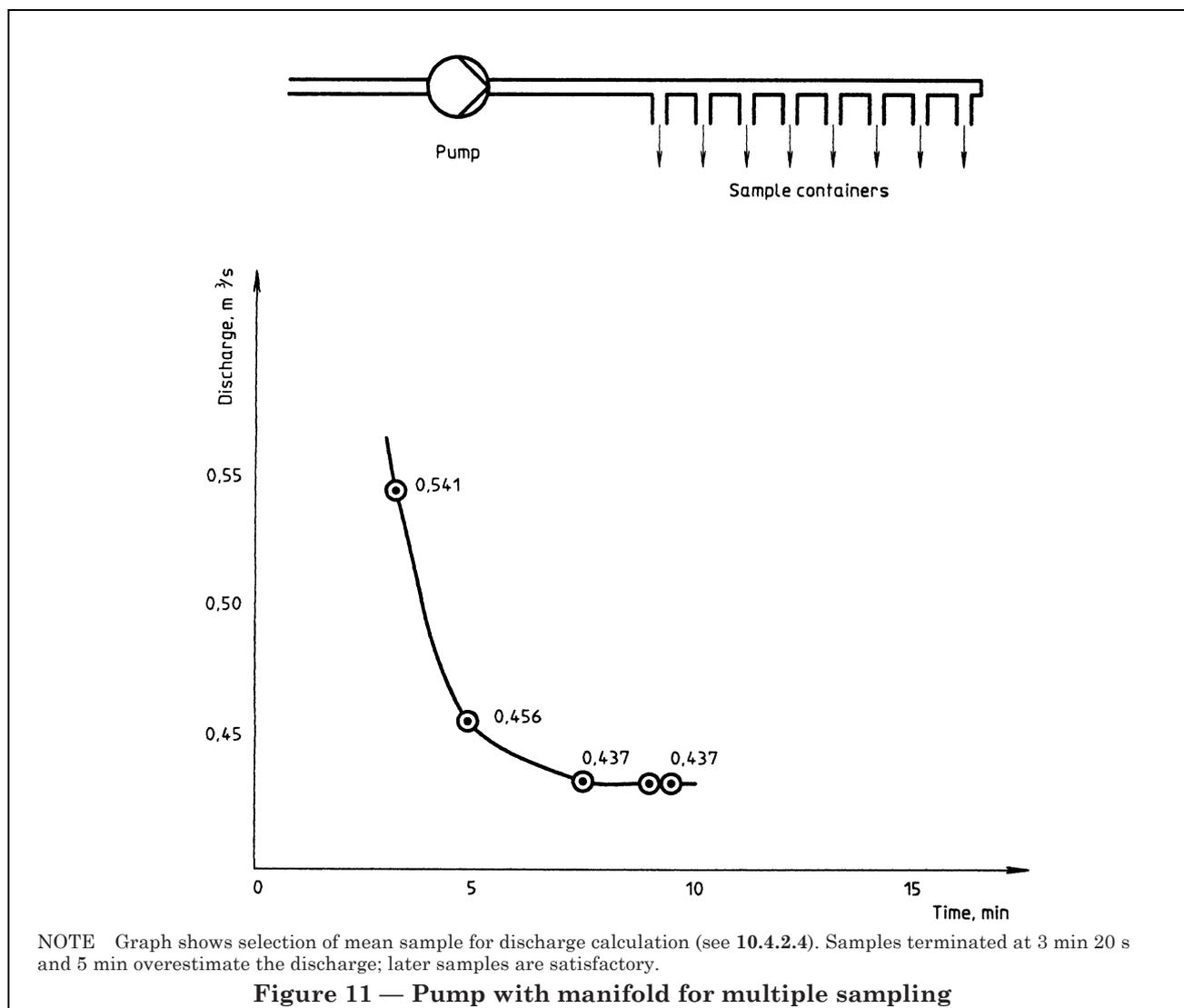


a) Several mean samples beginning at the same instant (see 10.4.2.4)



b) Several successive mean samples (see 10.4.2.5)

Figure 10 — Obtaining several mean samples



### 11.3 Sampling and analysis of results

In general, for the method of sudden injection, sampling is more complex than for the constant rate method. This is due to the necessity to measure mean values of concentration instead of the quasi-instantaneous values obtained on a concentration plateau. Sampling for the sudden injection method requires either the taking of a large number of samples on a rigid time schedule, or the use of sophisticated apparatus. The analysis of results by statistical methods is identical for the two methods but in general the constant rate method will supply a larger number of data, permitting greater accuracy in the final determination of the discharge.

## 12 Estimation of uncertainties

### 12.1 General

The accuracy of a discharge measurement is best expressed in terms of a statistically determined range of uncertainty. In this instance the measured discharge is the discharge calculated from the dilution of a known amount of tracer, and the uncertainty of the measurement is the range within which the true discharge can be expected, with 95 % probability, to lie (95 % confidence level).

The uncertainty of a measurement is estimated as the combination of uncertainties in the contributing sources of error. Thus, the relative influence of each contributing source can be assessed to determine whether, with the resources and techniques available, discharge can be measured with sufficient accuracy for the purpose in hand.

### 12.2 Uncertainties due to different kinds of error

Errors, leading to uncertainty in the discharge computation, are classified as random or systematic. Random errors are experimental errors or reading errors which lead to repeated measurements of a quantity being distributed about their mean or the unknown true value in accordance with the laws of probability. Systematic errors stem from inaccuracies inherent in the equipment and conditions of measurement; in the presence of systematic error, measurements of a quantity can be expected to be consistently higher or lower than the true value.

The uncertainty due to random errors can be estimated statistically in terms of the standard deviation. The standard deviation  $s_Y$  of  $n$  measurements of a variable  $Y$  is given by the equation:

$$s_Y = \left[ \frac{1}{n-1} \sum_{i=1}^n (Y_i - \bar{Y})^2 \right]^{1/2} \quad \dots(6)$$

where  $\bar{Y}$  is the arithmetic mean of  $n$  measurements.

The standard deviation of the mean is

$$s_{\bar{Y}} = s_Y / \sqrt{n} \quad \dots(7)$$

If the number of measurements of the quantity  $Y$  is large enough for the deviations from the mean to approach a normal distribution about zero, the uncertainty of the mean is approximately equal to  $2s_{\bar{Y}}$  at the 95 % confidence level.

It follows that the range in the value of the measured quantity is  $\bar{Y} \pm 2s_{\bar{Y}}$ .

From equations (6) and (7) it is evident that the range of uncertainty due to random errors can be reduced by increasing the number of measurements.

Because a measurement of a quantity subject to systematic error gives a biased estimate of the true value, the uncertainty due to systematic error cannot be reduced by increasing the number of measurements, but must be estimated subjectively on the basis of knowledge of the equipment and techniques involved. The difficulties inherent in the calculation of systematic errors are such that all efforts must be made to detect and reduce systematic errors to the minimum by suitable choice of equipment and techniques.

### 12.3 Sources of systematic error

The following five sources of systematic error, peculiar to dilution methods, are of particular interest:

#### 12.3.1 Error associated with the tracer

The tracer which is introduced into the injection cross-section is likely, in certain cases, to react with the water flowing in the channel, with suspended sediment, vegetation or the bed and banks of the channel or with the walls of sampling vessels in such a way as to reduce the concentration of tracer detected in the samples. This would lead to an overestimate of the discharge, the relative error being approximately equal to the relative loss of tracer.

#### 12.3.2 Error associated with the duration of the gauging

For gauging by the constant rate injection method, it is essential that the concentration of tracer at the sampling cross-section attains a constant value. For gauging by the sudden injection method, it is essential that the whole of the tracer cloud passes the sampling cross-section during the period of sampling. Retention of water in stagnant zones and in the boundary layer is not always negligible, and the time taken to achieve these conditions can be longer than would be expected. Cessation of sampling before these conditions are fulfilled results in an overestimation of the discharge, the relative error being similar in magnitude to the relative underestimation of the area of the tracer concentration—time curve in the sudden injection method, or of the plateau concentration in the constant rate injection method.

### 12.3.3 Error associated with poor mixing in the gauging reach

Use of the definition of good mixing (see 3.13) assumes that, owing to the turbulence of the flow, the concentration of tracer, though having a random statistical uncertainty, is independent of the position of the sampling point. If this is the case, there is no systematic error and the random uncertainty in the mean concentration can be calculated.

If the concentration, or the integral of the concentration—time curve, presents a systematic variation over the cross-section, the interaction of this distribution with that of velocity in the cross-section results in a difference between the mean of the sample concentrations and the true mean concentration of tracer in the cross-section. A positive or negative systematic error results. The magnitude of this error is dependent on the degree of mixing and on the distribution of discharge across the section, and it can be estimated only if detailed knowledge of the spatial distributions of tracer concentration and longitudinal velocity is available, but, expressed as a percentage, it may exceed twice the value of the expression  $(100 - x)$ , where  $x$  is the degree of mixing.

### 12.3.4 Error associated with change in storage volume in the reach

If the discharge changes appreciably during the measurement, systematic errors either positive or negative are introduced by the change in storage volume between the injection and sampling cross-sections. The error is least when the discharge varies smoothly and slowly, as for example on the recession limb of a hydrograph, and when the mean transit time between injection and sampling cross-sections is least. It is therefore advantageous to use a measuring reach that exceeds the mixing length by as small a margin as possible.

For the constant rate injection method, the relative error is approximately equal to the relative change in discharge over a period of time  $t^*$ , equal to the mean time of travel through the gauging reach, the error being positive for a falling discharge, and negative for a rising discharge. Similar errors are incurred with the sudden injection method, if the discharge measurement is assumed to relate to a time  $t^*$  after injection.

### 12.3.5 Error associated with sampling and analysis of samples

Errors are also likely to arise in determining the concentration of tracer in the sampling cross-section and in samples taken from this section. These may be due to sampling techniques, to an interaction between the walls of the sampling vessel and the tracer, or to systematic errors in the techniques used for analysis. Errors may also arise during the process of dilution of the injection solution. In particular, the calibration of volumetric glassware should be verified by gravimetric methods. Negative or positive errors lead to over- or underestimation respectively of the discharge.

## 12.4 Estimation of systematic uncertainty

The effects of systematic error may be incurred in the analysis of dilution gauging results by making certain assumptions about the distribution of the errors. As stated in 12.3, systematic error tends to lead to an overestimation of the discharge, but this is not always the case. The following procedure, which provides an estimate of the systematic component of the total uncertainty, may be used to assess the effect of the various errors outlined in 12.3, if sufficient information is available.

- a) For each source of systematic error, estimate the range of possible values of the discharge ( $Q_1$ ,  $Q_2$ ). For instance, if a loss by adsorption of between 2 % and 5 % of tracer is expected during the gauging, the systematic error arising from tracer will be between 2 % and 5 % in magnitude, and will be of positive sign. For a calculated discharge  $Q$  of 100 m<sup>3</sup>/s, the range of possible values of the actual discharge is from 95 m<sup>3</sup>/s to 98 m<sup>3</sup>/s.
- b) Apply a correction to the calculated value of discharge, to centre it within the range of possible values, i.e.  $Q = (Q_1 + Q_2)/2$ .
- c) Associated with this source of error there will be a remaining uncertainty of  $\pm (Q_2 - Q_1)/2$ , which can be combined with the estimated random uncertainty using the method in 12.6.

## 12.5 Computation of random uncertainties

The standard deviation of a quantity  $F$ , which is computed as a function of several quantities  $\bar{x}_1, \bar{x}_2, \dots$ , etc., each of which in turn is derived from the results of a number of measurements, can be determined by the combination of the standard deviations of  $\bar{x}_1, \bar{x}_2, \dots$ , etc. according to the rule:

$$s_F = (F_1^2 s_1^2 + F_2^2 s_2^2 + \dots)^{1/2}$$

where

$F_i$  is the partial derivative of  $F$  with respect to  $\bar{x}_i$ ;

$s_i$  is the standard deviation of  $x_i$ .

The general form of the dilution gauging equation (see 6.1)

$$Q = kD$$

is the product of several quantities, each of which has its own sources of uncertainty. Each of these contributing uncertainties should be assessed separately, and then combined to give the uncertainty in the discharge  $Q$ .

### 12.5.1 Constant rate injection method

The discharge is calculated from the equation:

$$Q = q^D$$

The standard deviation of  $Q$  is calculated from:

$$\left(\frac{s_Q}{Q}\right)^2 = \left(\frac{s_q}{q}\right)^2 + \left(\frac{s_D}{D}\right)^2 \quad \dots(8)$$

where

$s_q$  is the standard deviation of the injection rate  $q$ ;

$s_D$  is the standard deviation of the dilution factor  $D$ .

The standard deviation  $s_q$  is derived either from a statistical regression analysis, e.g. of the water level in a cylindrical or prismatic vessel against time, or from a repetition of injections in the laboratory and the application of equations (6) and (7).

The standard deviation of the dilution factor is calculated either from a series of repetitive dilutions of the injected solution, or from the following equation:

$$s_D = \left( \frac{s_{c'_1}^2 D^2}{c_1'^2} + \frac{s_{c'_2}^2 D^2}{c_2'^2} + \frac{s_d^2 c_1'^2}{c_2'^2} \right)^{1/2} \quad \dots(9)$$

where the standard deviations  $S_{c'_1}$  and  $S_{c'_2}$  are evaluated from repetitive analysis (see 6.2) and  $s_d$  is the standard deviation of the dilution process, estimated from the known accuracy of the dilution equipment, e.g. glassware.

### 12.5.2 Sudden injection method

The discharge is calculated from the equation

$$Q = \frac{V}{T_p} D$$

The standard deviation of  $Q$  is calculated from

$$s_Q = \left( \frac{s_V^2 Q^2}{V^2} + \frac{s_{T_p}^2 Q^2}{T_p^2} + \frac{s_D^2 Q^2}{D^2} \right)^{1/2} \quad \dots(10)$$

The standard deviation  $s_V$  is calculated either from the known accuracy of the vessel used to determine  $V$ , or from repetitive laboratory measurement, by gravimetric methods for example. The standard deviation,  $s_{T_p}$  is determined from the known accuracy of the timing device used. In the simplest case, that of measurement of an integrated river sample, the determination of  $s_D$  follows the procedure used in 12.5.1.

Where discrete samples are used, the concentration  $\bar{c}_2$  is determined as a sum of  $n$  quantities  $c_{2,i}$  at each sample point:

$$\bar{c}_2 = a_1 c_{2,1} + a_2 c_{2,2} + \dots + a_n c_{2,n}$$

where  $a_i$  are coefficients depending on the time interval to which each sample relates. In the case of equal time intervals, the coefficients  $a_i$  are all equal to  $1/n$ . The standard deviation of  $\bar{c}_2$  is determined by:

$$s_{\bar{c}_2} = (s_1^2 a_1^2 + s_2^2 a_2^2 + \dots + s_n^2 a_n^2)^{1/2}$$

where  $s_i$  is adopted here as a simplified convention for  $s_{c_{2,i}}$ , the standard deviation associated with each sample, calculated by repeated determination.

To each sample  $\bar{c}_2$  there now corresponds a standard deviation  $S_{\bar{c}_2}$ .

However, it is likely that this "intrasample" standard deviation will be exceeded by the "intersample" standard deviation obtained by applying equations (6) and (7) to  $\bar{c}_2$ . This "intersample", standard deviation arises from poor mixing and other causes associated with the problems of sampling in the field.

If the "intersample" standard deviation is calculated by:

$$s_b = \left[ \frac{1}{m} \times \frac{1}{m-1} \sum_{i=1}^m (\bar{c}_{2,i} - \bar{\bar{c}}_2)^2 \right]^{1/2}$$

where

$\bar{\bar{c}}_2$  is the mean of the  $\bar{c}_{2,i}$ ;

$m$  is the number of sampling points in the sampling cross-section,

then an approximation to the standard deviation of  $\bar{c}_2$  is given by:

$$s_{\bar{c}_2} = \left( s_b^2 + \frac{1}{m} \sum_{i=1}^m s_{\bar{c}_{2,i}}^2 \right)^{1/2}$$

The standard deviation of the dilution factor is calculated as in equation (9), using  $s_{\bar{c}_2}$  in place of  $s_{c_2}$ .

### 12.5.3 General note

The nature of many of the combination formulae given in 12.5, that of a sum of two or more squares of quantities, results in the expressions for the standard deviations being dominated by the largest terms in the sums. For example, the standard deviation of a product arises largely from the least precisely determined term in the product. It is useful to compare the relative standard deviations  $s_{\bar{Y}}/\bar{Y}$  of the terms that comprise the expression for the discharge: substantial improvement in the final accuracy of the discharge measurement can only be achieved by improving the accuracy of the term that most contributes to the error.

For example, if the 95 % confidence limits on the injection rate and the dilution factor for a constant rate injection gauging are  $\pm 5\%$  and  $\pm 1\%$  respectively, the 95 % confidence limits on the discharge will be  $\pm 5,099\%$ . Halving the error in the injection rate will reduce the final uncertainty to  $\pm 2,693\%$ , while halving the error in the dilution factor will still leave an uncertainty in the discharge of  $\pm 5,025\%$ .

**12.6 Computation of the total uncertainty**

The uncertainties arising from random and systematic sources are combined according to the following rule, to give confidence limits on the final result. If the systematic uncertainties arising from the various sources, calculated according to the method in 12.4, are given by  $\pm X_1, \pm X_2, \dots, \pm X_k$  and the 95 % confidence limits on the discharge arising from random uncertainty are  $\pm 2s_Q$ , the total uncertainty of the measurement is

$$\pm (X_1^2 + X_2^2 + \dots + X_k^2 + 4s_Q^2)^{1/2} \dots(11)$$

**12.7 Working examples of computation of uncertainties**

The following case histories are offered as examples of the methods of computation of the discharge and its uncertainty.

NOTE 10 The actual numerical values used should not be taken by the user as an indication of the relative magnitude of the uncertainties involved in a gauging by the dilution method.

**12.7.1 Example 1 — Constant rate injection method**

Tracer was injected from a reservoir whose liquid level was measured on a scale of centimetres (e.g. a Mariotte vessel with a sight tube). The injection vessel calibration factor was determined in the laboratory by repeated tests, and this factor was used to convert scale readings to litres of injection solution.

To preserve a degree of generality in the working example, it is assumed that the tracer concentrations were measured on equipment whose output was in the form of a scale reading between zero and 100, i.e. as a percentage of full scale deflection (f.s.d). Also in the interests of generality, the random uncertainty associated with the analytical equipment is regarded as an element of the uncertainty in the mean sample concentrations, and is not treated explicitly.

Readings of the vessel scale against time were used in a linear regression to calculate the injection rate:

Time			Scale reading
h	min	s	cm
11	48	57	35,0
12	09	16	31,0
12	17	10	29,5
12	35	30	26,0
12	40	46	25,0
13	01	34	21,0
13	27	28	16,0
13	53	03	11,0
14	08	18	8,0
14	28	19	4,0
14	46	03	0,5

Gradient:  $- 3,238 7 \times 10^{-3}$  cm/s

Standard deviation of gradient:  $7,034 9 \times 10^{-6}$

Calibration factor: 1,039 7 l/cm

Standard deviation of calibration factor:

$$4,531 2 \times 10^{-4} \text{ l/cm}$$

The injection rate,  $q$ , is the product of the gradient times the calibration factor:

$$q = 3,238 7 \times 1,039 7 \times 10^{-3} \text{ l/s}$$

$$q = 3,367 3 \times 10^{-3} \text{ l/s}$$

The standard deviation of the injection rate,  $s_q'$  is

$$s_q = (7,034 \ 9^2 \times 10^{-12} \times 1,039 \ 7^2 + 4,531 \ 2^2 \times 10^{-8} \times 3,238 \ 7^2 \times 10^{-6})^{1/2}$$

$$= 7,460 \ 0 \times 10^{-6} \text{ l/s}$$

(= 0,222 % of injection rate)

Estimated discharge: 80 l/s

Estimated dilution factor

$$D' = 80 / (3,367 \ 3 \times 10^{-3}) = 23 \ 760$$

Injected solution diluted by:

12 500; 15 000; 20 000; 25 000; 30 000; 40 000; 50 000

The standard deviations of each of these dilutions, which were made by pipette and flask, can be calculated from the accuracy specified for the glassware. For example for the dilution of 20 000, which was made using in succession a 50 ml pipette into a 500 ml flask, a 10 ml pipette into a 1 000 ml flask, then a 25 ml pipette into a 500 ml flask, assuming that 95 % confidence limits for the pipette volumes were  $\pm 0,2$  % and for the flask volumes were  $\pm 0,05$  %, the standard deviation  $s_D$  of the dilution process is given by

$$s_D = D \left( s_{V_{p1}}^2 / V_{p1}^2 + s_{V_{f1}}^2 / V_{f1}^2 + \dots + s_{V_{p3}}^2 / V_{p3}^2 \right)^{1/2}$$

where the pipette volumes are  $V_{pi}$  and the flask volumes are  $V_{fi}$ .

For each term of this expression, it is assumed that the quoted confidence limits are plus or minus twice the standard deviation, so that for a flask  $S_{V_f} / V_f$  is 0,000 25 and for a pipette  $S_{V_p} / V_p$  is 0,001.

The standard deviation

$$s_D = 20 \ 000 \ (3 \times 0,000 \ 25^2 + 3 \times 0,001^2)^{1/2}$$

$$= 35,707 \ 2 \ (\text{= } 0,150 \ \% \text{ of the dilution factor})$$

Another contribution to the uncertainty in the dilution factor  $D$  is due to the scatter in the river samples, measured in the standard dilution method as relative dilutions of the injected solution in the form

$$c_r = \frac{1}{D}$$

where  $c_r$  is the relative concentration of injected solution.

If the readings from the analytical equipment for the standard dilutions are:

Dilutions	Relative concentration	Reading (% f.s.d)
12 500	$80 \times 10^{-6}$	93,0
15 000	$66,7 \times 10^{-6}$	77,3
20 000	$50 \times 10^{-6}$	57,7
25 000	$40 \times 10^{-6}$	47,1
30 000	$33,3 \times 10^{-6}$	38,9
40 000	$25 \times 10^{-6}$	28,8
50 000	$20 \times 10^{-6}$	23,3

and the readings associated with the samples taken are:

Right bank	Centre	Left bank
45,3	47,4	45,8
43,9	46,4	45,5
46,0	45,4	46,6

where three sets of samples were taken at times  $t_1$ ,  $t_2$  and  $t_3$ , then the associated relative concentrations (read from a plotted calibration curve) are:

Right bank	Centre	Left bank	Mean
$39,0 \times 10^{-6}$	$40,8 \times 10^{-6}$	$39,4 \times 10^{-6}$	$39,7 \times 10^{-6}$
$37,8 \times 10^{-6}$	$39,9 \times 10^{-6}$	$39,1 \times 10^{-6}$	$38,9 \times 10^{-6}$
$39,6 \times 10^{-6}$	$39,1 \times 10^{-6}$	$40,1 \times 10^{-6}$	$39,6 \times 10^{-6}$
Mean $38,8 \times 10^{-6}$	$39,9 \times 10^{-6}$	$39,5 \times 10^{-6}$	$39,4 \times 10^{-6}$

The dilutions corresponding to the samples are:

Right bank	Centre	Left bank	Mean
25 641	24 510	25 381	25 177
26 455	25 063	25 575	25 698
25 253	25 575	24 938	25 255
Mean 25 783	25 049	25 298	25 377

The mean dilution factor is 25 377 and the standard deviation of the dilution factor [from equation (7)] is 181,392 (see Annex C) which shows that the nine samples can be treated as independent estimates of  $c_2$ .

Combining the uncertainties in the dilution factor gives:

$$s_D = (35,707^2 + 181,392^2)^{1/2} = 184,873$$

or  $s_D = 0,729\%$  of the dilution factor.

The discharge,  $Q$ , is evaluated from the equation:

$$Q = qD = 3,3673 \times 10^{-3} \times 25\,377 = 85,45 \text{ l/s}$$

and the standard deviation,  $s_Q$ , from the equation (8):

$$s_Q = 85,45 \left[ \frac{(7,4600 \times 10^{-6})^2}{(3,3673 \times 10^{-3})^2} + \frac{184,873^2}{25\,377^2} \right]^{1/2} = 0,651 \text{ l/s}$$

or  $s_Q = 0,761\%$  of the discharge.

The 95 % confidence limits are therefore:

$$\pm 2 \times 0,651 = \pm 1,302 \text{ l/s}$$

or 1,524 % of the discharge.

The degree of mixing  $x$  may be calculated from the formula given in 8.1.1 b):

$$x = 100 \left[ 1 - \frac{(39,4 - 38,8) + (39,9 - 39,4) + (39,5 - 39,4)}{2 \times 3 \times 39,4} \right]$$

$$= 99,5 \%$$

As an example of the computation of the systematic uncertainty, from 12.3.3, it is seen that the systematic error arising from poor mixing can be  $2(100 - x)$ . Thus the systematic error, assuming for this case that this is an upper limit, and in the absence of information about the distribution of discharge across the stream, could be  $\pm 1,0 \%$ . As this error is systematic, no correction is applied to the calculated value of discharge, but account must be taken of the systematic uncertainty  $X''_Q = \pm 1,0 \%$  when computing the total uncertainty,  $X_Q$ , of the discharge measurement.

Records from a stage hydrograph indicated that there had been a decrease in stream discharge of approximately  $0,75 \%$  per hour during the measurement. The mean time of travel through the gauging reach was  $0,5$  h. From 12.3.4, the systematic error in the discharge measurement from this cause could be expected to be  $+0,375 \%$ , but uncertainties in the measurement of the time of travel and the rate of change of discharge mean that this error could lie between  $+0,28 \%$  and  $+0,47 \%$ . The correction to be applied to the discharge estimate is:  $-(0,28 + 0,47)/2$ , or  $-0,375 \%$ , resulting in a corrected discharge figure of

$$85,45 \left( 1 - \frac{0,375}{100} \right) = 85,13 \text{ l/s}$$

and a remaining systematic uncertainty of  $\pm (0,47 - 0,28)/2$ , (or  $\pm 0,095 \%$ ).

The total uncertainty is

$$X_Q = \pm \left[ \left( \frac{1,0 \times 85,13}{100} \right)^2 + \left( \frac{0,095 \times 85,13}{100} \right)^2 + 4 \times 0,651^2 \right]^{1/2}$$

$$= \pm 1,558 \text{ l/s}$$

or  $1,83 \%$  of discharge.

The result may therefore be presented as follows:

a) Discharge

$$Q = 85,13 \text{ l/s} \pm 1,83 \%$$

Random uncertainty

$$X'_Q = \pm 1,52 \% (95 \% \text{ confidence level})$$

or

b) Discharge

$$Q = 85,13 \text{ l/s}$$

Random uncertainty

$$X'_Q = \pm 1,52 \% (95 \% \text{ confidence level})$$

Systematic uncertainty

$$X''_Q = \pm 1,0 \%$$

A comparison of the magnitudes of the relative standard deviations of the elements in the calculation of discharge, expressed here in percentages, demonstrates that the primary source of uncertainty in the discharge is the scatter of samples taken from the stream. This may be under the control of the operator: for example a longer gauging reach could improve mixing and a larger number of samples could reduce the scatter from other causes.

### 12.7.2 Example 2 — Sudden injection method

The concentrated solution of tracer was injected into the current by means of an ampoule which was weighed before and after injection.

The volume injected was  $3,862\ 0\ \text{cm}^3$  and the standard deviation for this quantity, taking into account the precision of the balance, was  $0,000\ 25\ \text{cm}^3$ .

Discrete samples were taken every two minutes at three points along the sampling section. The samples were mixed so as to obtain a single sample for each sampling point, corresponding to a period  $T_0 = 80\ \text{min}$ . The standard deviation for the time period, calculated according to the precision of the clock, was very low and thus negligible (see 12.5.3).

The samples were analysed on an instrument with a linear response (e.g. a counter, for radioactive tracers). Three subsamples were analysed for each sampling point as well as for water without tracer, taken upstream of the injection point, diluted by a factor of  $6,052\ 2 \times 10^{10}$ .

The results, expressed as concentrations, are given as follows:

	Sample			Mean of three samples	Standard deviation
	1	2	3		
Background noise $c_0$	2,37	2,36	2,41	2,38	0,015 3
Diluted injection solution $c'_1 + c_0$	80,98	81,41	80,67	81,02	0,214 2
River samples					
Right bank: $c_{21} + c_0$	14,48	14,29	14,67	14,48	0,109 7
Centre: $c_{22} + c_0$	14,74	14,47	14,70	14,64	0,084 1
Left bank: $c_{23} + c_0$	15,26	14,86	15,05	15,06	0,115,5
			Mean	14,73	

The standard deviations are calculated according to equations (6) and (7).

After subtracting the background noise, the mean concentrations  $\bar{c}'_1$  and  $\bar{c}_{2i}$  are:

$$\bar{c}'_1 = 81,02 - 2,38 = 78,64$$

$$\bar{c}_{21} = 14,48 - 2,38 = 12,10$$

$$\bar{c}_{22} = 14,64 - 2,38 = 12,26$$

$$\bar{c}_{23} = 15,06 - 2,38 = 12,68$$

$$\bar{c}_2 = 14,73 - 2,38 = 12,35 \quad (\text{mean})$$

The corresponding standard deviations are:

$$s_{c'_1} = (0,214 \ 2^2 + 0,015 \ 3^2)^{1/2}$$

$$= 0,214 \ 7$$

or 0,273 % of  $c'_1$ .

$$s_{c_{21}} = (0,109 \ 7^2 + 0,015 \ 3^2)^{1/2}$$

$$= 0,110 \ 8$$

or 0,916 % of  $c_{21}$  (right).

$$s_{c_{22}} = (0,084 \ 1^2 + 0,015 \ 3^2)^{1/2}$$

$$= 0,085 \ 5$$

or 0,697 % of  $c_{22}$  (centre).

$$s_{c_{23}} = (0,115 \ 5^2 + 0,015 \ 3^2)^{1/2}$$

$$= 0,116 \ 5$$

or 0,919 % of  $c_{23}$  (left).

The degree of mixing may be calculated from the formula given in 8.1.1 a):

$$x = 100 \left\{ 1 - \left[ \frac{(12,35 - 12,10)}{2 \times 3 \times 12,35} + \frac{(12,35 - 12,24) + (12,68 - 12,35)}{2 \times 3 \times 12,35} \right] \right\}$$

$$= 99,1 \ %$$

The standard deviation of the dilution factor is evaluated from a consideration of the known precision of the balance used for the gravimetric dilutions:

First dilution

The dilution factor for 0,321 6 g of injection solution in a flask of 998,3 g capacity is:

$$D_1 = 998,3/0,321 \ 6$$

$$= 3 \ 104,17$$

The standard deviation for  $D_1$ , given that the syringe used for the dilution can be weighed to a 95 % confidence limit of  $\pm 0,000 \ 14$  g and the flask can be weighed to a 95 % confidence limit of  $\pm 0,20$  g, is:

$$s_{D_1} = 3 \ 104,17 (0,000 \ 07^2/0,321 \ 6^2 + 0,1^2/998,3^2)^{1/2}$$

$$= 0,743 \ 8$$

Second dilution

The dilution factor for 0,317 1 g of injection solution from the first dilution in a flask of 997,8 g capacity is:

$$D_2 = 997,8/0,317 \ 1$$

$$= 3 \ 146,64$$

The standard deviation for  $D_2$ , making the same assumptions as above, is:

$$s_{D_2} = 3\,146,64 \left( 0,000\,07^2 / 0,317\,1^2 + 0,1^2 / 997,8^2 \right)^{1/2}$$

$$= 0,762\,9$$

Third dilution

The dilution factor for 0,161 1 g of injection solution from the second dilution in a flask of 998,2 g capacity is:

$$D_3 = 998,2 / 0,161\,1$$

$$= 6\,196,15$$

The standard deviation for  $D_3$  is thus:

$$s_{D_3} = 6\,196,15 \left( 0,000\,07^2 / 0,161\,1^2 + 0,1^2 / 998,2^2 \right)^{1/2}$$

$$= 2,762\,9$$

The combined dilution factor is:

$$D = D_1 \times D_2 \times D_3$$

$$D = 3\,104,17 \times 3\,146,64 \times 6\,196,14$$

$$= 6,052\,2 \times 10^{10}$$

The standard deviation of the combined dilution factor is:

$$s_D = D \left( \frac{s_{D_1}^2}{D_1^2} + \frac{s_{D_2}^2}{D_2^2} + \frac{s_{D_3}^2}{D_3^2} \right)^{1/2}$$

$$s_D = 6,052\,2 \times 10^{10} \left( 0,743\,8^2 / 3\,104,17^2 + \right.$$

$$\left. + 0,762\,9^2 / 3\,146,64^2 + \right.$$

$$\left. + 2,762\,9^2 / 6\,196,15^2 \right)^{1/2}$$

$$= 3,396\,9 \times 10^7$$

or 0,056 % of the total dilution factor  $D$ .

The discharge, calculated using only the river sample from the centre, is:

$$Q = \frac{V}{T_0} \times D$$

$$= \frac{3,862\,0 \times 10^{-3} \times 6,052\,2 \times 10^{10} \times 78,64}{80 \times 60 \times 12,26}$$

$$= 312\,347 \text{ l/s}$$

$$= 312 \text{ m}^3/\text{s}$$

The standard deviation of the discharge is:

$$\begin{aligned}
 s_Q &= 312\,347 \left( \frac{0,000\,25^2}{3,862\,0^2} + \frac{3,396\,9^2 \times 10^{14}}{6,052\,2^2 \times 10^{20}} + \right. \\
 &\quad \left. + \frac{0,214\,7^2}{78,64^2} + \frac{0,085\,5^2}{12,26^2} \right)^{1/2} \\
 &= 2\,345,9 \text{ l/s} \\
 &= 2,35 \text{ m}^3/\text{s}
 \end{aligned}$$

or 0,753 % of  $Q$ .

By similar methods, it is possible to compute values of the discharge  $Q$  and its standard deviation,  $s_Q$ , corresponding to the samples from the right and left banks:

$$Q = 316\,477 \text{ l/s}$$

$$s_Q = 3\,029,3 \text{ l/s (right bank)}$$

$$Q = 302\,001 \text{ l/s}$$

$$s_Q = 2\,899,6 \text{ l/s (left bank)}$$

Analysis of variance (see Annex C) shows that in this case the variation of concentration with sampling point is significant, so the nine samples cannot be combined to give a realistic estimate of the uncertainty of  $c_2$  by the normal rules, i.e. by treating the measurements as independent estimates of the same quantity. It is thus necessary to use the method outlined in 12.5.2, and to proceed as if the three values of  $c_2$  had been obtained by the numerical integration of a number of discrete samples.

The "intersample" standard deviation  $s_b$  is thus calculated as:

$$\begin{aligned}
 s_b &= \left[ \frac{1}{m} \times \frac{1}{m-1} \sum_{i=1}^m (\bar{c}_{2i} - \bar{\bar{c}}_2)^2 \right]^{1/2} \\
 &= \left\{ \frac{1}{3} \times \frac{1}{2} \left[ (12,10 - 12,35)^2 + (12,26 - 12,35)^2 + \right. \right. \\
 &\quad \left. \left. + (12,68 - 12,35)^2 \right] \right\}^{1/2} \\
 &= 0,173\,0
 \end{aligned}$$

Hence:

$$\begin{aligned}
 s_{\bar{c}_2} &= \left( 0,173\,0^2 + \frac{0,110\,8^2 + 0,085\,5^2 + 0,116\,5^2}{3} \right)^{1/2} \\
 &= 0,202\,4
 \end{aligned}$$

or 1,64 % of  $\bar{\bar{c}}_2$ .

In the calculation of the discharge and its standard deviation, replacing  $s_{c_2}$  by  $s_{\bar{c}_2}$  calculated as above, and  $c_2$  for the individual sampling points by the mean  $\bar{c}_2$  results in:

$$Q = \frac{3,862\ 0 \times 10^{-3} \times 6,052\ 2 \times 10^{10} \times 78,64}{80 \times 60 \times 12,35}$$

$$= 310\ 071\ \text{l/s}$$

$$= 310,1\ \text{m}^3/\text{s}$$

and

$$s_Q = 310\ 071 \left( \frac{0,000\ 25^2}{3,862\ 0^2} + \frac{3,396\ 9^2 \times 10^{14}}{6,052\ 2^2 \times 10^{20}} + \frac{0,214\ 7^2}{78,64^2} + \frac{0,202\ 4^2}{12,35^2} \right)^{1/2}$$

$$= 5\ 154,66\ \text{l/s}$$

$$= 5,15\ \text{m}^3/\text{s}$$

or 1,66 % of  $Q$ .

The 95 % confidence limits associated with the random uncertainty are therefore:

$$\pm 2 \times 5,15$$

$$= \pm 10,3\ \text{m}^3/\text{s}$$

or  $\pm 3,32$  % of  $Q$ .

As an example of the computation of the systematic uncertainty, from **12.3.3**, it is seen that the systematic error arising from poor mixing can be  $2(100 - x)$ . Assuming for this case that this is an upper limit, and in the absence of information about the distribution of discharge across the stream, the systematic error could thus be  $\pm 1,8$  %. As this systematic error is symmetrical, no correction is applied to the calculated value of discharge, but the systematic uncertainty  $X''_Q = \pm 1,8$  % must be taken into account when computing the total uncertainty of the measurement.

The total uncertainty is:

$$X_Q = \pm \left[ \left( \frac{1,8 \times 310,1}{100} \right)^2 + 4 \times 5,15^2 \right]^{1/2}$$

$$= \pm 11,72\ \text{m}^3/\text{s}$$

or  $\pm 3,78$  % of the discharge  $Q$ .

The result may therefore be presented as follows:

a) Discharge

$$Q = 310,1\ \text{m}^3/\text{s} \pm 3,78\ \%$$

Random uncertainty

$$X'_Q = \pm 3,32\ \% \text{ (95 \% confidence level),}$$

or

b) Discharge

$$Q = 310,1\ \text{m}^3/\text{s}$$

Random uncertainty

$$X'_Q = \pm 3,32\ \% \text{ (95 \% confidence level).}$$

Systematic uncertainty

$$X''_Q = \pm 1,8\ \%$$

## Annex A (normative) Standard dilutions using pipettes and flasks

### A.1 Recommended minimum equipment

For the preparation of samples:

- 20 beakers of 100 ml capacity
- 2 sets of pipettes of 20 ml and 50 ml capacity

For the preparation of standard dilutions:

- 2 sets of pipettes of 10 ml, 20 ml, 25 ml and 50 ml capacity
- 2 sets of volumetric flasks of 250 ml, 500 ml, 1 000 ml and 2 000 ml capacity
- 10 beakers or flasks of 250 ml capacity

### A.2 Table for the preparation of standard dilutions

	1/800 to 1/12 500 ( $1,25 \times 10^{-3}$ to $8 \times 10^{-5}$ )		1/8 000 to 1/125 000 ( $1,25 \times 10^{-4}$ to $8 \times 10^{-6}$ )		1/80 000 to 1/1 250 000 ( $1,25 \times 10^{-5}$ to $8 \times 10^{-7}$ )		1/800 000 to 1/12 500 000 ( $1,25 \times 10^{-6}$ to $8 \times 10^{-8}$ )	
	Initial solution		50/500		10/1 000		10/1 000 × 50/500	
	10/1 000	25/2 000	10/1 000	25/2 000	10/1 000	25/2 000	10/1 000	25/2 000
$\frac{25}{250}$		1/800 ( $1,25 \times 10^{-3}$ )		1/8 000 ( $1,25 \times 10^{-4}$ )		1/80 000 ( $1,25 \times 10^{-5}$ )		1/800 000 ( $1,25 \times 10^{-6}$ )
$\frac{25}{250}$	1/1 000 ( $1 \times 10^{-3}$ )		1/10 000 ( $1 \times 10^{-4}$ )		1/100 000 ( $1 \times 10^{-5}$ )		1/1 000 000 ( $1 \times 10^{-6}$ )	
$\frac{20}{250}$	1/1 250 ( $8 \times 10^{-4}$ )		1/12 500 ( $8 \times 10^{-5}$ )		1/125 000 ( $8 \times 10^{-6}$ )		1/1 250 000 ( $8 \times 10^{-7}$ )	
$\frac{25}{500}$		1/1 600 ( $6,25 \times 10^{-4}$ )		1/16 000 ( $6,25 \times 10^{-5}$ )		1/160 000 ( $6,25 \times 10^{-6}$ )		1/1 600 000 ( $6,25 \times 10^{-7}$ )
$\frac{25}{500}$	1/2 000 ( $5 \times 10^{-4}$ )		1/20 000 ( $5 \times 10^{-5}$ )		1/200 000 ( $5 \times 10^{-6}$ )		1/2 000 000 ( $5 \times 10^{-7}$ )	
$\frac{20}{500}$	1/2 500 ( $4 \times 10^{-4}$ )		1/25 000 ( $4 \times 10^{-5}$ )		1/250 000 ( $4 \times 10^{-6}$ )		1/2 500 000 ( $4 \times 10^{-7}$ )	
$\frac{25}{1 000}$		1/3 200 ( $3,125 \times 10^{-4}$ )		1/32 000 ( $3,125 \times 10^{-5}$ )		1/320 000 ( $3,125 \times 10^{-6}$ )		1/3 200 000 ( $3,125 \times 10^{-7}$ )
$\frac{25}{1 000}$	1/4 000 ( $2,5 \times 10^{-4}$ )		1/40 000 ( $2,5 \times 10^{-5}$ )		1/400 000 ( $2,5 \times 10^{-6}$ )		1/4 000 000 ( $2,5 \times 10^{-7}$ )	
$\frac{20}{1 000}$	1/5 000 ( $2 \times 10^{-4}$ )		1/50 000 ( $2 \times 10^{-5}$ )		1/500 000 ( $2 \times 10^{-6}$ )		1/5 000 000 ( $2 \times 10^{-7}$ )	
$\frac{25}{2 000}$		1/6 400 ( $1,563 \times 10^{-4}$ )		1/64 000 ( $1,563 \times 10^{-5}$ )		1/640 000 ( $1,563 \times 10^{-6}$ )		1/6 400 000 ( $1,563 \times 10^{-7}$ )
$\frac{25}{2 000}$	1/8 000 ( $1,25 \times 10^{-4}$ )		1/80 000 ( $1,25 \times 10^{-5}$ )		1/800 000 ( $1,25 \times 10^{-6}$ )		1/8 000 000 ( $1,25 \times 10^{-7}$ )	
$\frac{20}{2 000}$	1/10 000 ( $1 \times 10^{-4}$ )		1/100 000 ( $1 \times 10^{-5}$ )		1/1 000 000 ( $1 \times 10^{-6}$ )		1/10 000 000 ( $1 \times 10^{-7}$ )	
$\frac{20}{250} \times \frac{20}{250}$		1/12 500 ( $8 \times 10^{-5}$ )		1/125 000 ( $8 \times 10^{-6}$ )		1/1 250 000 ( $8 \times 10^{-7}$ )		1/12 500 000 ( $8 \times 10^{-8}$ )

**EXAMPLES**

To obtain a dilution of 1/16 000, first select the range of dilutions at the top of the table (1/8 000 to 1/125 000). Following the relevant columns of the table downwards, the initial solution should be diluted by 50/500 (50 ml pipette into 500 ml flask), then by 25/2 000 (25 ml pipette into 2 000 ml flask) and finally by 25/500 (25 ml pipette into 500 ml flask). A dilution of 1/5 000 000 would require four successive dilution stages: 10/1 000, 50/500, 10/1 000 and 20/1 000.

**Annex B (normative)****Standard dilution ranges obtained with a micrometer-type microburette****B.1 Characteristics**

Volume: 0,5 ml

Volume delivered for 1 turn of the micrometer: 0,01 ml

Total number of turns: 50, each turn subdivided into 50 graduations

Absolute accuracy:  $\pm 0,000\ 05$  ml, or  $\pm 0,5\%$  for 0,01 ml.

Estimated dilution factor:  $D' = 20\ 000$  to  $2\ 000\ 000$

Estimated relative concentration:  $\frac{1}{D'} = 5 \times 10^{-5}$  to  $5 \times 10^{-7}$

Final volume obtained for each standard dilution: 50 ml

**B.2 Procedure**

- a) Determine, according to the measurement conditions, the estimated dilution factor  $D' = Q'/q$  (constant rate injection method) or  $D' = Q'T_p/V$  (sudden injection method).
- b) Select in column 1 of Table B.1 the range containing  $D'$ , which defines the range of standard dilutions to be obtained.
- c) Perform the predilution operation or operations defined in columns 2 and 3 by means of the pipettes and flasks indicated (e.g. 25/1 000 denotes the use of a 25 ml pipette and a 1 000 ml flask).
- d) Fill the barrel of the microburette with the solution obtained by predilution.
- e) Prepare eight clean dry beakers, and pipette into each beaker 50 ml of background water. One beaker, marked "0", is reserved for a blank sample.
- f) Using the number  $N_0$  in column 4 to enter the second table, the number  $N$  of turns required for each standard dilution is obtained. The range of standard dilutions is obtained by delivering into the seven remaining beakers the quantity defined by the number  $N$  of micrometer turns.

NOTE 11 The ranges have been calculated in such a way as to require only a single filling of the barrel of the microburette, i.e. the total number of turns is less than 50.

NOTE 12 Each range includes seven dilutions which are evenly distributed about the mid-value (given by  $N_0$ ) which is close to  $D'$ , according to the range chosen; the extreme values obtained correspond approximately to  $2D'$  and  $D'/2$ .

NOTE 13 Glassware required for the predilutions is limited to a single series of pipettes and flasks.

Table B.1 — Preparation of standard dilution ranges using a micrometer-type microburette

Ranges of estimated dilution D'	Predilution		Mid-value $N_0$	Relative concentration $\times 10^{-6}$	
	Pipette/flask	Pipette/flask		for $N_0$ turns	for 1 turn
1	2	3	4	5	6
18 000 to 22 000	—	50/1 000	5	50	10
22 000 to 28 000	—	50/1 000	4	40	10
28 000 to 36 000	—	25/1 000	6	30	5
36 000 to 45 000	—	25/1 000	5	25	5
45 000 to 56 000	—	25/1 000	4	20	5
56 000 to 72 000	—	25/2 000	6	15	2,5
72 000 to 90 000	—	25/2 000	5	12,5	2,5
90 000 to 110 000	—	10/1 000	5	10	2
110 000 to 140 000	—	10/1 000	4	8	2
140 000 to 180 000	—	10/2 000	6	6	1
180 000 to 220 000	—	10/2 000	5	5	1
220 000 to 280 000	—	10/2 000	4	4	1
280 000 to 360 000	25/500	50/1 000	6	3	0,5
360 000 to 440 000	25/500	50/1 000	5	2,5	0,5
440 000 to 560 000	25/500	50/1 000	4	2	0,5
560 000 to 720 000	25/500	50/2 000	6	1,5	0,25
720 000 to 900 000	25/500	50/2 000	5	1,25	0,25
900 000 to 1 120 000	25/500	50/2 000	4	1	0,25
1 120 000 to 1 440 000	50/1 000	25/2 000	6	0,75	0,125
1 440 000 to 1 800 000	50/1 000	25/2 000	5	0,625	0,125
1 800 000 to 2 500 000	50/1 000	25/2 000	4	0,5	0,125

Table B.2 — Micrometer turns for ranges with mid-values  $N_0$

Mid-value $N_0$	Number of turns N to be made for each dilution								
	4	2	2,5	3	4	5	6	8	—
5	—	2,5	3	4	5	6	8	10	—
6	—	—	3	4	5	6	8	10	12

## Annex C (normative)

### Analysis for variance as a test of non-randomness

Frequently river samples are taken in a systematic manner to provide for example a set of samples at selected times and positions in the sampling cross-section. During the analysis samples may also be subjected to repeat determinations. The use of these measurements to determine the uncertainty of the discharge measurement requires a clear understanding of which samples may legitimately be combined in the calculation. Two sets of data are taken from the working example in 12.5, to illustrate the use of analysis of variance techniques in this context.

#### C.1 Example 1

In a gauging by the constant rate injection method, samples were taken from the stream at three points in the sampling cross-section and at three times. The results are given in terms of instrument readings:

Table C.1 is used to construct Table C.2 of sums of squares.

Between-times sum of squares:

$$(138,2^2 + 135,8^2)/3 - 412,3^2/9 \\ = 1,365\ 23$$

Between-positions sum of squares:

$$(135,2^2 + 139,2^2 + 137,9^2)/3 - 412,3^2/9 \\ = 2,757\ 81$$

Total sum of squares:

$$(45,3^2 + 47,4^2 + 45,8^2 + \dots + 46,6^2) - 412,3^2/9 \\ = 7,707\ 03$$

Table C.1 — Instrument readings

	Right bank	Centre	Left bank	Sum
Time 1	45,3	47,4	45,8	138,5
Time 2	43,9	46,4	45,5	135,8
Time 3	46,0	45,4	46,6	138,0
Sum	135,2	139,2	137,9	412,3

Table C.2 — Sums of squares of values from Table C.1

Source	Sum of squares	Degrees of freedom	Variance estimate	Variance ratio
Position	2,757 81	2	1,378 91	1,538 97
Time	1,365 23	2	1,682 615	0,761 85
Residual (by subtraction)	3,583 99	4	0,895 997	
Total	7,707 03	8		

For  $m$  positions and  $n$  times, the degrees of freedom are  $m - 1$  and  $n - 1$  respectively. The total degrees of freedom are  $mn - 1$ . Entries in the “variance estimate” column are obtained by dividing the sum of squares by the degrees of freedom. Entries in the “residual” row are obtained by subtraction from the “total” row. A comparison between the “variance ratio” entries in the table (obtained as the ratio of the variance estimate and the residual variance) with a table of the Snedecor  $F$  values, with the appropriate choice of degrees of freedom, shows that the variation with time and position is not significant. The samples may safely be combined on the assumption that they represent independent estimates of the same random variable.

**C.2 Example 2**

In a gauging by the sudden injection method, samples were prepared by the combination of a large number of discrete samples. Three composite samples were obtained, from the right bank, centre and left bank. Each sample was analysed three times in the laboratory. It may be assumed that the repeat measurements represent random samplings of the same quantity: the question here is whether the variation with position across the sampling cross-section is significant or not. The results, given in terms of instrument readings, are presented in Table C.3.

**Table C.3 — Instrument readings**

	Right bank	Centre	Left bank	Sum
Sample 1	14,48	14,74	15,26	44,48
Sample 2	14,29	14,47	14,86	43,62
Sample 3	14,67	14,70	15,05	44,42
Sum	43,44	43,91	45,17	132,52

Table C.3 is used to construct Table C.4 of sums of squares.

Between-positions sum of squares:

$$(43,44^2 + 43,91^2 + 45,17^2)/3 - 132,52^2/9 \\ = 0,534\ 668$$

Total sum of squares:

$$(14,48^2 + 14,74^2 + \dots + 14,70^2 + 15,05^2) - \\ - 132,52^2/9 = 0,729\ 981$$

**Table C.4 — Sum of squares of values from Table C.3**

Source	Sum of squares	Degrees of freedom	Variance estimate	Variance ratio
Position	0,534 668	2	0,267 334	8,212 52
Residual	0,195 313	6	0,032 552	—
Total	0,729 981	8		

A comparison between the variance ratio and a table of the Snedecor  $F$  values shows that the variation across the stream is significant at the 2,5 % level, i.e. the three samples should not be regarded as independent estimates of the same quantity.

## Annex D (informative)

### Bibliography

[1] ISO 5168:—, *Measurement of fluid flow — Evaluation of uncertainties*<sup>2)</sup>.

[2] ISO 8363:1986, *Liquid flow measurement in open channels — General guidelines for the selection of methods*.

<sup>2)</sup> To be published. (Revision of ISO 5168:1978)

## List of references

See national foreword.

**BS 3680-2A:  
1995  
ISO 9555-1:  
1994**

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